by Einstein's theory of relativity, further research on the synthetic geometry of hyperspace may be expected. Doubtless central projection and section will still be a guiding principle or, at least, a very useful concept.

2. The Steiner Program. The Steiner program appeared in the *Systematischen Entwickelung* ten years after the publication of Poncelet's great work. Its basic ideas are:

1. Definition of primitive forms (point-row, sheaf of rays or pencil of rays, sheaf of planes, etc.).
2. Postulation of ideal elements and the notion of the perspective position of two primitive forms.
3. Definition of projectively related primitive forms by means of chains of perspectivity (followed by Cremona 40 years later).
4. Generation of new forms by means of projectively related primitive forms.

In this program, the invariance of the cross-ratio of corresponding quadruples of elements in projectively related primitive forms appears as a theorem. The use of this invariant as a definition of projective relationship is due to Chasles (*Aperçu Historique*, 1837).

By means of this program Steiner and his co-workers and followers added a vast number of geometric forms to those previously known and derived many of the properties of these forms. The whole field thus opened has been explored both by synthetic and by analytic methods. But it would be wrong to suppose that all the details of this very general program have been filled in. One may suggest, perhaps, as an example a discussion of rational curves. Thus two cubic equations:

$$\lambda^3 U + \lambda^2 V + \lambda W + Z = 0,$$
$$\lambda^3 A + \lambda^2 B + \lambda C + D = 0,$$

where the coefficients of the powers of λ are linear functions of the coördinates of a point in a plane, represent two projectively related envelopes of rays of the third class. These envelopes generate a rational sextic curve in the plane. That is, the two equations have at least one root in common provided their eliminant vanishes. This eliminant is of sixth degree in the coördinates of the point and thus represents a sextic curve. This curve is rational since the coördinates of any point on it are immediately expressible as rational functions of λ. The two equations will have two roots in common, and thus the curve have a double point, provided the coördinates of the point cause the determinants of the matrix

$$\begin{Vmatrix} U & V & W & Z & 0 \\ 0 & U & V & W & Z \\ A & B & C & D & 0 \\ 0 & A & B & C & D \end{Vmatrix}$$

to vanish simultaneously. The number of common solutions is 10 which may, of

course, be combined in various ways according to the assumptions made about the coefficients of λ in the original equations. An analytic discussion would involve the theory of restricted equations. A synthetic discussion would involve a study of certain subsidiary loci whose common points are singular points on the rational curve generated by the projectively related envelopes. We touch closely here the work initiated by Hesse (1866) and carried forward by Klein, Franz Meyer, Haase and others.

We know some things about the location of singular points on a proper rational curve but much remains unknown. It would seem feasible, in the way suggested, to add to our knowledge of configurations of points in a plane that can be multiple points on proper rational curves of orders higher than 5. The method, too, is capable of extension to higher space. Thus, two projectively related sheaves of planes of second class generate a ruled surface of fourth order etc.

One may mention also the application of the Steiner program to the study of non-euclidean geometry, in particular to those representations of non-euclidean geometry in Euclidean space. No difficulty would be encountered in carrying out the Steiner program on a sphere which, with proper restrictions, can be taken as a Euclidean picture of the elliptic plane, but I know of no analogous study of the geometry on a surface of constant negative curvature as the Euclidean picture of geometry in the hyperbolic plane. And I feel very sure that much remains to be done in projective non-euclidean geometry of hyperspace which might very well be accomplished along the lines of the Steiner program.

3. The Von Staudt Program. Von Staudt added three general features to the Steiner program (*Beiträge*, 1847):

1. The definition of projective relationship by means of harmonic division, thus removing the cumbersome chain of perspectivity as a definition from the Steiner program and measurement as a fundamental notion from the Chasles definition.

2. The introduction of the notion of the polarity in the plane and in space and defining conics and quadric surfaces as loci of self conjugate elements in polarities. Imaginary conics and quadric surfaces appear for the first time defined by means of uniform polarities in the plane and in space.

3. A geometric definition of imaginary points, lines, and planes by means of elliptic involutions on the sustaining form.

In regard to this program, the consideration of double polarities in the plane and in space seems capable of further extension. Thus, synthetically, two polarities in a bundle of rays, one of which is the orthogonal polarity, lead most easily to the properties of cones of the second order. Two polarities in a plane lead to quadric transformations and hence, by repetition, to Cremona transformations. Analytically this amounts to a study of simultaneous symmetric bilinear forms. In space of an odd number of dimensions, skew-symmetric bilinear forms would have to be considered.

But Von Staudt's great achievement was the geometric definition of imaginary

elements. The necessity for such a definition had been recognized by Poncelet, Steiner, Chasles and others but no satisfactory synthetic definition existed before 1847. In spite of this, Von Staudt's work bore little fruit for many years probably because of the formal style in which it appeared. It remained for Reye (*Geometrie der Lage*, 1866) to introduce Von Staudt and his work to mathematicians in general and to geometers in particular.

In 1872, Klein proposed a simplification of Von Staudt's definition of imaginary elements. Two conjugate imaginary points on a straight line, for example, are represented by a cyclic involution of order three on the line. Thus, synthetically,

$$ABC \curlywedge BCA \curlywedge CAB$$

represents one imaginary point on the line ABC, and

$$CBA \curlywedge BAC \curlywedge ACB$$

represents the conjugate imaginary point on the same line.

Analytically, if λ and μ are the coördinates of two real points on a line, the projectivity

$$\frac{\mu - z}{\mu - \bar{z}} = \omega \frac{\lambda - z}{\lambda - \bar{z}}; \qquad \omega^3 = 1, \qquad \omega \neq 1, \tag{1}$$

is cyclic and of order three and has the conjugate imaginary points z and $\bar{z}$ as its double points. The projectivity *defines* these imaginary points.

For $z, \bar{z} = x \pm iy$, the projectivity reduces to

$$\mu = \frac{(\sqrt{3}x - y)\lambda - \sqrt{3}(x^2 + y^2)}{\sqrt{3}\lambda - (\sqrt{3}x + y)} \tag{2}$$

and furnishes the bond between the Klein representation of imaginary points and the familiar representation by means of the Wessel-Argand diagram or the Gauss-plane.

The simpler involution

$$\mu = \frac{\lambda x - (x^2 + y^2)}{\lambda - x} \tag{3}$$

has exactly the same double points (Doppelpunktsinvolution; involutione unita) and furnishes the bond between the Von Staudt elliptic involution representation and the Gauss-plane.

Ramorino has given a complete account of theories about imaginary numbers and their geometric representations (*Giorn. di Mat.*, 1897–98).

Von Staudt also arrived at the notion of chains of imaginary points. Given any three points, real or imaginary, the locus of a fourth point such that the cross-ratio of the four points is a given real number is a chain. When imaginary points on a line are pictured as real points in a Gauss-plane, chains are represented

as straight lines or circles in the Gauss-plane. While much was previously known about the geometry of lines and circles in a plane, the Von Staudt chain was a strange new thing in geometry.

4. The Segre Program. Segre did much for geometry in several directions. Had he been asked, he would doubtless have called himself an algebraic geometrician. The program I am attributing to him deals with imaginary geometry, or more properly, the geometry of imaginary elements. This work was initiated by him in papers appearing in *Atti di Torino*, 1889–90–91, and in *Math. Ann.*, 1890. Here we meet the hyperalgebraic forms; *i.e.*, algebraic forms in the real components of an imaginary element. Or, what comes to the same thing, algebraic forms in the coördinates of conjugate imaginary elements. Thus forms in z, $\bar{z}$ (Eq. 1) or in x, y (Eqs. 2 or 3) are hyperalgebraic forms. Hermitian forms are examples. The geometric representation of such forms leads to the study of hyperalgebraic manifolds. Consider imaginary points in a plane. Each point has four real components and four-dimensioned space is required to form real pictures of geometric relations in the plane. Three equations connecting the real components of an imaginary point are the equations of a *thread*. Von Staudt's chains are particular threads. All imaginary points whose real components satisfy two equations are said to lie on a membrane or tissue. A single equation in the real components is the equation of a tri-dimensioned manifold. Segre's article in the *Math. Ann.* is concerned in the main with the real representations of hyperalgebraic manifolds in hyperspace.

Hyperalgebraic manifolds of an odd number of dimensions are entirely new entities in geometry. Segre also studied the linear transformations amongst the elements of hyperalgebraic manifolds and so was lead to anti-projectivities, anti-collineations, anti-polarities, etc. In an anti-projectivity, for example, the cross-ratios of corresponding quadruples of elements have conjugate imaginary values. In the study of linear transformation in the hyperalgebraic domain, Segre was anticipated somewhat, although unknown to him, by the Danish geometer C. S. Juel. Juel's thesis, published at Copenhagen 1885, deals with certain linear transformations in the hyperalgebraic domain which he called "symmetralities." This thesis was later published in *Acta Mathematica*, 1890.

Analysis in the hyperalgebraic domain leads to the definition and study of hypercomplex numbers and thus touches closely the work of Weierstrass, *Gött. Nach.*, 1884, Study in various papers, and others.

Thus is revealed an astonishing wealth of detail—a wealth beyond the dreams of the most avaricious geometer. The postulation of ideal points, the introduction of irrational points as limiting points in infinite series constructable by harmonic division (Dedekind postulate), and the geometric definition of imaginary points make synthetic geometry co-extensive with the algebra of n-ary forms; but it is doubtful if any of the great spirits who created and perfected the algebra of quantics ever dreamed that the prosaic symbols with which they wrought could hide such a wealth of geometric forms as has been uncovered along the lines

suggested by Segre. In witness to this remark one has only to read Coolidge's *Complex Geometry* (1924) and the papers by Coolidge, Young, Graustein, and others in this country. No one can doubt that there is much yet to be discovered in this field.

5. The Klein Program. Perhaps the most far-reaching and fruitful program for geometry was that of Felix Klein (*Erlanger Program*, 1872). In this program, Klein proposed to classify geometries by means of groups of transformations. Real projective geometry appears as the study of the invariant properties of geometric forms under the group of real linear point transformations. By extending the group to include reciprocal transformations—*i.e.*, point to line, point to plane, or point to hyperplane—projective geometry is made to include dual configurations and their properties.

The general group of projective transformations contains various subgroups. Those projective transformations which leave unchanged the ideal line in a plane (the ideal R_{n-1} in n-space) form a group to which belongs affine geometry. Affine geometry has come into prominence recently through the writings of Weyl and others in connection with the theory of relativity. Those projective transformations which leave invariant the circular points in any plane form a group to which belongs elementary geometry including the modern geometry of the triangle and its related circles.

Other subgroups have been considered which leave invariant certain loci. For example, those projective transformations which leave unchanged a proper conic (real or imaginary) form a group to which belongs plane non-euclidean geometry.

The projective group is capable of further extension by including within it anti-projectivities, anti-collineations, etc., thus including the Segre geometry of imaginary elements within the projective fold. To connect this field with that opened up by Study in *Theorie der Dynamen* and in various papers appears to me to be a problem of considerable attractiveness. And the synthetic study of continuous groups of projective transformations, begun by Newson in 1895 and carried on by Emch and others, cannot be regarded as complete.

The work of Enriques-Fano (*Ann. di Mat.*, 1897) shows that finite continuous groups of Cremona transformations in the plane and in three-space are equivalent to projective transformations in hyperspace. Thus, in space of a sufficiently great number of dimensions, all geometries admitting finite continuous groups of transformations may be classed as projective. Unquestionably many details remain to be filled-in in this comprehensive program: details concerning the inter-relations, isomorphisms, etc., among the various finite continuous groups of transformations contained within the $(n^2 - 1)$-parametered group of projective transformations in n-space.

What has been said thus far applies to entire space (Gesammtraum) and to finite continuous groups of transformations amongst the elements of this space. But there exist infinite continuous groups of point-transformations in this space;

e.g., the group of all analytic point-transformations. This group has no proper invariant other than the dimensions of the space. If, however, we consider only the infinitesimal neighborhood of a point, the differentials of the coördinates of the point undergo a projective transformation by virtue of the group of analytic transformations we are considering. Here we enter the domain of projective differential geometry and become interested in projective differential invariants—a field so well cultivated by Wilczynski, Green, Lane and others. Here we meet the Christoffel-Lipschitz theory of quadratic differential forms which forms the mathematical basis for the modern theory of relativity. A promising recent addition to this field is the geometry of paths initiated by Veblen and Thomas.

And what can be said about the geometry of imaginary points in the neighborhood of any given point, real or imaginary? Is there a complex projective differential geometry, or a projective differential geometry of imaginary elements? A few scattered theorems and notions, perhaps, and little more.

In conclusion, one may say that what has been called the "Segre program" together with the theory of groups of transformations is the most promising field for further research. This field is broad, is relatively new, and almost every issue of a mathematical magazine bears witness to its fecundity.

It has been said that projective geometry is a finished subject—all very pretty and beautiful but lacking in inspiration because completed. Nothing could be further from the truth. No young enthusiast seeking adventure in mathematical fields need avoid projective geometry because of fancied sterility. If he has the strength of purpose to persevere until he can read understandingly the various articles on geometry in the *Encyklopædia*, he must feel as every traveler feels when, after struggling up the Alps, he sees all Italy lie before him.

THE USE OF COMPUTING RODS IN CHINA.

By DAVID CHIN-TE CHENG, Hinghwa City, Fukien, China.

The use of computing rods was a matter of daily importance in early times in China. We have no means of knowing just who invented the system nor at what date, but in early Chinese literature we find it stated that computers were trained in the art during a period of ten years; and that the use of computing rods was looked upon as fundamental in the solving of problems. For instance, it was asserted that the *K'iu-ch'ang Suan-shu*[1] (Arithmetic in Nine Sections), which probably was written by Li Shou[2] during the reign of Huang-ti, the Yellow Emperor, who began his reign in the year 2704 B.C.,[3] could be easily understood if

[1] *K'iu-ch'ang Suan-shu* or *Chiu-chang Suan-shu*, Smith, *History of Mathematics*, vol. I, pp. 31–33; Smith and Mikami, *History of Japanese Mathematics*, pp. 11–13; Cajori, *History of Mathematics*, p. 71.

[2] Wu Ch'eng Ch'uan, *Kang Chien I Chih Lu* (Brief History of China, monograph 1, p. 4; Yuan Yuan, *Ch'ou Jen Chuan* (Biography of Chinese Mathematicians), monograph 1, p. 1.

[3] Smith, *History of Mathematics*, vol. I, p. 24.

the system was known in which purple rods were used for positive numbers, and black for negative. Some writers claim that this system was used even as early as 2704 B.C. In the *Chou-pei Suan-king*,[4] probably written about 1105 B.C., Shang Kao states that "the process of numbers came from circles and squares; circle came from square, square came from squaring rule, and squaring rule came from the numbers nine times nine, which is equal to eighty-one." [5] "During the time of Huan Kung of Ch'i in the Chou dynasty" (about 680 B.C.), so Mei Fu, a writer of the same period asserts, "the numbers nine and nine were used." [6] The numbers "nine and nine" meant the numbers from one to nine, and a recognition of the product nine times nine equals eighty-one. The fact that the computing rods were based upon the numbers from one to nine led certain writers to assert definitely that the people during that time used them in solving problems. The *Sun Tze Suan Ching*, written by Sun Tze about 350 B.C., used this system in multiplication, division, and squaring. The computing rods were known c. 542 B.C.[7] and are referred to as counting stalks in a statement of Hiao-tze, the ruler of Ts'in from 361 to 337 B.C. They are mentioned again about 215 B.C., and some specimens of this period were displayed in a museum of the Emperor Ngan (397–419). These were known as arrow rods and were eighty in number. They were about 18 inches long, some made of bone and others of horn. The reason for using such long rods was that the early inhabitants of China performed their computations on the ground. In the reign of Wu-ti (140–37 B.C.) of the Han dynasty, it is related that an astronomer Sang Hung (about 113 B.C.) was very skillful in his use of the rods. In the third century of our era it is recorded that Wang Jung, a minister of state, spent his nights in reckoning his income with ivory calculating rods, and the expression "to reckon with ivory rods" is still used as an allusion to wealth. In the time of the Emperor Ch'eng (326–343) the counting rods were made of wood, ivory, iron, bamboo, or paper, and two centuries later the Emperor Siuen Wu (500–516) had counting rods cast in iron for the use of the people.[8] After the Han dynasty it became the custom to compute on a table, and shorter rods were therefore used; these being about four American inches long and one tenth of an inch thick. The fact is that the early mathematics in China, until the Sung and Yuan dynasties (before 1368), was based entirely upon the use of computing rods; but during the Ming dynasty (after 1368) the system lost its significance entirely on account of the use of the abacus (*suan pan* [9]) and the introduction of European methods.

The historian Mei Wen Ting (1633–1721), in his work on *Li Suan Ch'uan Shu* (the chapter on *Ku Suan Ch'i K'ao*, a study of the ancient calculating instru-

[4] Smith, *History of Mathematics*, vol. I, pp. 29–31; Smith and Mikami, *History of Japanese Mathematics*, p. 9; Mikami, *The Development of Mathematics in China and Japan*, p. 4; Cajori, *History of Mathematics*, p. 71.

[5] Tai Chen, *Tse Suan* (Computing Rods), Introduction.

[6] Tai Chen, *Tse Suan* (Computing Rods), Introduction.

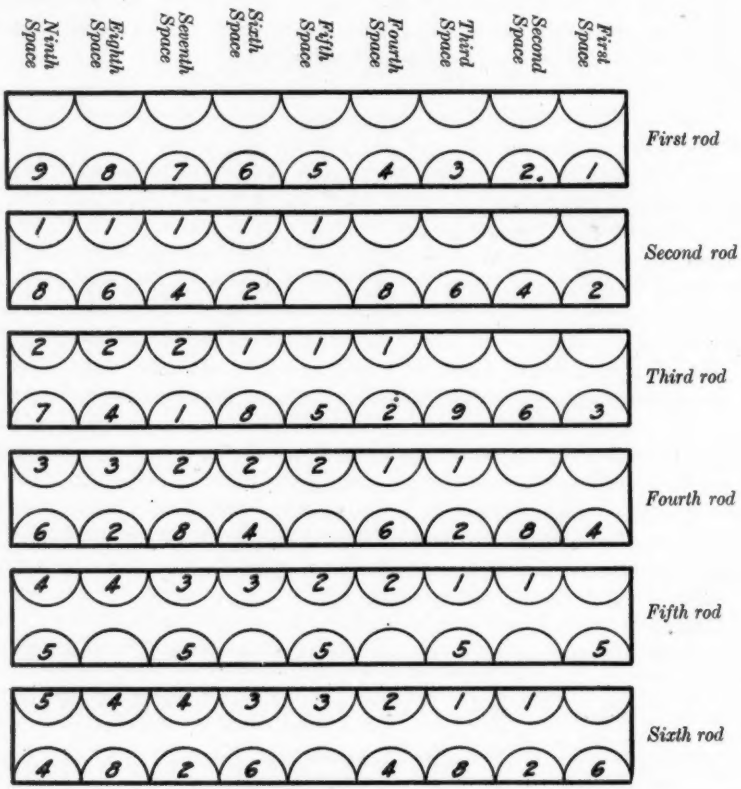
[7] Smith, *History of Mathematics*, vol. I, p. 96.

[8] Smith, *History of Mathematics*, vol. II, pp. 169–170.

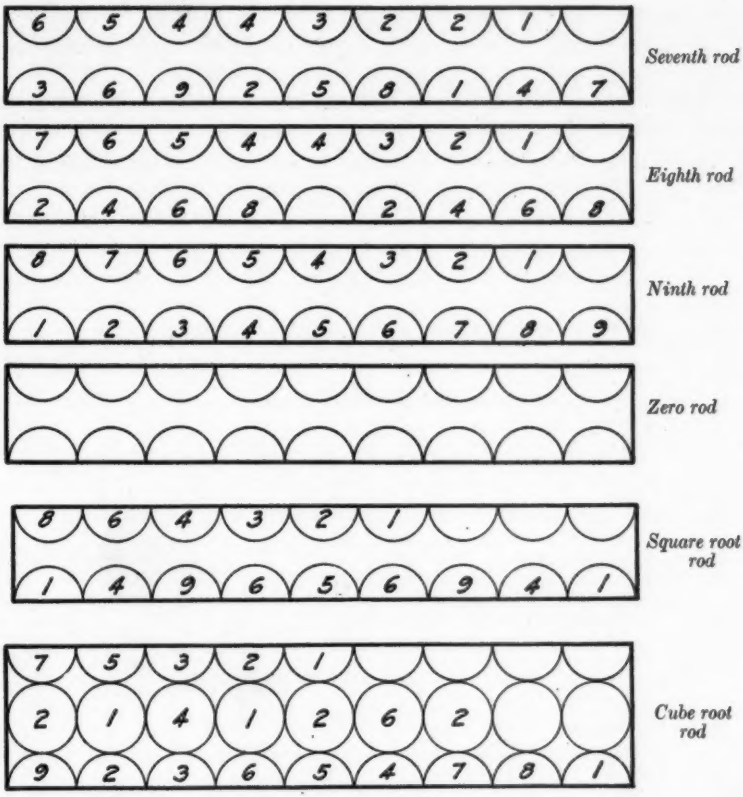
[9] Smith, *History of Mathematics*, vol. II, p. 168.

ments), states that about the beginning of the Christian era 271 rods constituted a set, or handful, and that they formed a hexagon that had nine rods on a side. This means that they were arranged in six groups of which the ends of each formed a triangular number of $1 + 2 + \cdots + 9$ units, or 45 in all. Six of these make 6×45, or 270, and these six were grouped about one central rod, making 271, thus affording an illustration of the use of figurate numbers in the East.[10]

We are not certain as to the nature of the very early rods, but we have accurate information as to the later ones. This comes to us from various standard works, a list of which is given in the bibliography at the end of this article. In the case of the later system, beginning about the thirteenth century, each rod was divided into nine spaces, and in each space there were drawn two semicircles, one above and one below. In the nine spaces on the rods the numbers are definitely arranged in the following way:



[10] Mei Wen Ting, *Ku Suan Ch'i K'ao* (Ancient calculating instrument); Smith, *History of Mathematics*, vol. II, p. 170.



Spaces on any rod are numbered from the right-hand end. When in the following discussion reference is made to the number of a rod, the number in the lower semicircle of the first space is meant, zero rod being the one in which that space is vacant. The square-root and cube-root rods are not numbered. To find the number in any space of the regular computing rods: multiply the number of the space by the number of the rod.

The numbers in the upper semicircles are considered as belonging to the tens' place, while those in the lower semicircles are considered as units. When two rods are used, the lower semicircle of the upper rod and the upper semicircle of the lower rod are taken to form one whole circle, and the numbers within each circle are added together so as to form the digit of that place. When two rods are used, three digits are thus formed; when three rods are used, four digits are formed, and so on. This will be clearly understood by reading problems numbered 1 and 2, which follow.

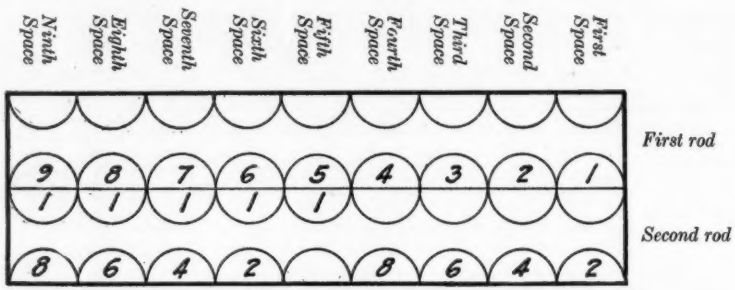
Computing rods are used in multiplication, division and squaring. Addition

and subtraction are the only prerequisites to the working with them. The selection of the rods to be used depends upon the figures to be employed in the operation.

Multiplication on computing rods is somewhat similar to the process in modern arithmetic, as is evident from the following examples:

Problem 1. Twelve tailors who are in the army each get 360 *chin* of rice. What is the total amount of rice given?

Solution: To multiply 360 by 12, the first and second rods are used.

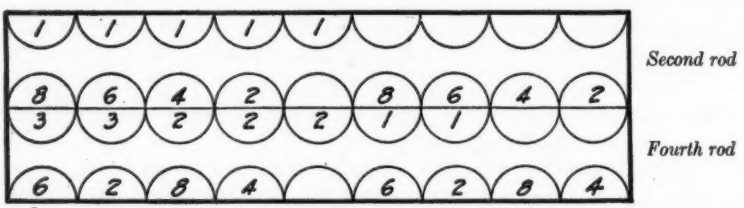


Assuming the operation which involves the 0, we first multiply the 6 in the 360 by 12. This is done by reading to the sixth space and obtaining 072. We next multiply the 3 by reading to the third space and getting as a result 036. Adding these two results together, we have 4320 as the final answer.

0	000
6	072
3	036
	4320

Problem 2. In *Fang t'ien* (squaring the farm) 1 *mu* is equal to 240 *pu*. How many *pu* are in 125 *mu*?

Solution: Using 240 as the multiplier, the second and fourth rods are used and an extra 0 is annexed to the result.



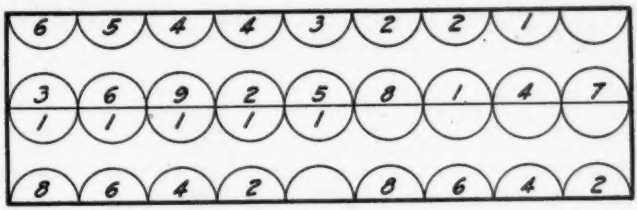
First multiply the 5 of 125, by reading to the fifth space and getting 120; next to multiply by 2, read to the second space and get 048; and finally to multiply by 1, read the first space, 024. Adding the three results, the answer is 30,000.

	000
5	120
2	048
1	024
	300000

The process of division by computing rods is just the opposite of that of multiplication, the quotient being read from the number of the space in which the figure most nearly approximates to the figure to be divided.

Problem 1. The sun moves 360 degrees in the year, which is divided into 72 equal parts. How many degrees are there in each part?

Solution: Since 360 is divided by 72, the seventh and second rods are used.



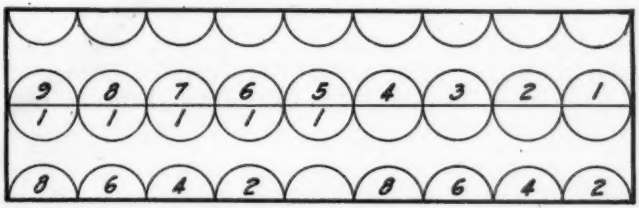
Seventh rod

Second rod

Read to the fifth space, the number is 360; subtracting, 360 − 360 = 0, and so 5 is the quotient.

Problem 2. 129,600 years is divided into 12 parts. How many years are there in each part?

Solution: Since 129,600 is divided by 12, the first and second rods are used.



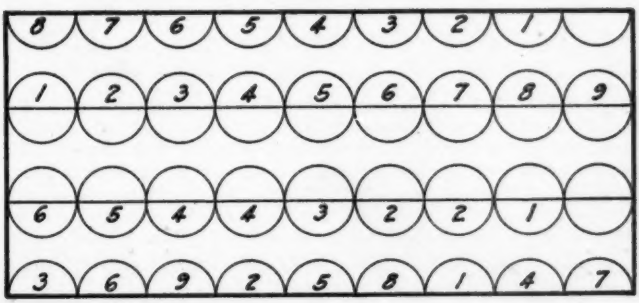
First rod

Second rod

Read 012 from the first space on the rod; subtracting, 129,600 − 120,000 = 9600, and so the first figure of the quotient is 1. Next read 096 from the eighth space; subtracting, 9600 − 9600 = 0, and so the second figure of the quotient is 8. Between the first and the second figure of the quotient there is one place vacant, which is filled with 0, and similarly with the last two places. The answer is 10,800.

Problem 3. 21,768 yards of cloth are divided among 907 people. How many yards of cloth does each person get?

Solution: Since 21,768 is divided by 907, the ninth, zero, and seventh rods are used.



Ninth rod

Zero rod

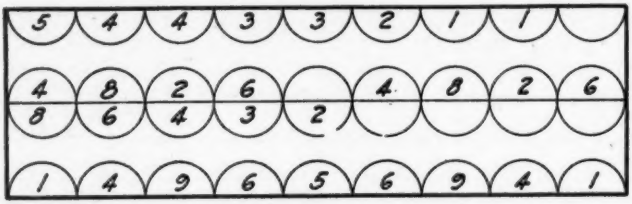
Seventh rod

Read 1814 from the second space, which is nearest to 2126; subtracting, 21,768 − 18,140 = 3628, and the first figure of the sum becomes 2; next read 3628 from the fourth space; subtracting, 3628 − 3628 = 0, and so the second figure of the quotient is 4. The answer is 24 yards.

According to Mei Wen Ting "the method of finding square root was used in the *Chou-peï Suan-king*, in which Shang Kao told Prince Chóu-kung that the use of the squaring rule was for the earth while the use of the circle was for the heavens." The use of the computing rod in finding square roots was a common practice in this period, and resembled closely our present system.

Problem: Find the square root of 129,600.

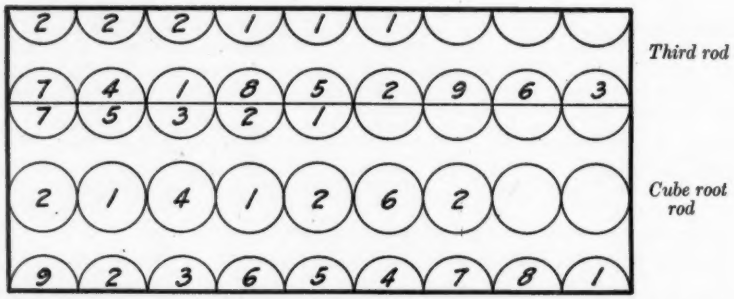
Solution: Mark off the number by placing a dot over every other place, beginning with units. Thus 12 is in the first left-hand period. Reading to the third space on the square-root rod we have 9, which is nearest to 12, and so the first figure of the root is 3, with a remainder of 3. Multiply the 3 in the root by 2, which gives 6, which shows that we are to use the sixth of the general computing rods together with the square-root rod.



Read to the sixth space on both of the rods, and the number is 396, which is exactly the next part in the number of which the square root is desired. Therefore the second figure of the root is 6, and, annexing zero, the final answer becomes 360.

Problem: Find the cube-root of 1331.

Solution: We first use the special cube-root rod, as follows:



In cube root mark off the periods by a dot over every third figure. Since 1 is the first figure of the number, the first space on the cube-root rod is read, giving 1. Subtracting this from the first period in the dividend there is no remainder. The first figure of the root is therefore 1. Now take 1 times 10, which is 10, and 10 times 10 times 3, which is 300; then using the third rod together with the cube-root rod, read from the first space the number is 301. Then 1 (first space) times 10 equals 30, and 301 + 30 = 331. Subtracting, 331 − 331 = 0, and thus the second figure in the root is 1. Therefore the final answer is 11.

Having now shown the nature of the work with this kind of computing rods, the question naturally arises as to the antiquity of the device. Evidently the

ancient rods referred to by Mei Wen Ting (1633–1721) were mere computing sticks such as were used to represent the coefficients of an equation. The statement that the later kind came into use about the thirteenth century is also due to Mei Wen Ting. He also says that the squaring rule was used in the *Chou-pei Suan-king*, and that the computing rod was used in finding the square root. That it was this later kind of rod is, however, very improbable, since no certain mention is made of this type until modern times. That these later rods were first used about the thirteenth century may be true, but they are described by no writers before Mei Wen Ting. This puts the description about the close of the seventeenth century, a hundred years after the Jesuit influence began in China, and more than fifty years after Napier's rods became generally known in the West. While the Chinese rods are by no means identical with those of Napier, there is a resemblance. It therefore becomes an interesting problem for us in China to ascertain (1) whether our rods were unquestionably used as early as the thirteenth century, as our later historians assert; and (2), if not, whether they were suggested by the Napier rods, which had no doubt reached our country through the Jesuit missionaries. In fact, Mei Wen Ting suggests this very possibility.

Bibliography.

Fang Chung T'ung (1687), *Shu Tu Yen*, a book treating of geometry, abacus, arithmetic, and computing rods. Reprinted in 1890 at Cheng-tu.

Mei Wen Ting, *Li Suan Ch'uan Shu*, Chapter on *Ku Suan Ch'i K'ao* (A study of the old mathematical instruments), 1723.

Mei Wen Ting, *Ch'ou Suan* (The computing rods). Written in 1723, but edited by his grandson and printed later.

Tai Chen (1744), *Tse Suan* (Computing rods), Shanghai, 1890.

Tan Wen, *Shu Hsueh Hsun Yuan* (To find the origin of mathematics), Chiu-chiang or Chang-sha, 1750.

Hsu Kuei Lin (1811), *Suan Yu* (Introduction to mathematics). Printed in 1830.

Lao Nai Hsuan, *Ku Ch'ou Suan K'ao Shih* (The old computing rods), 1886.

Lao Nai Hsuan, *Yen Yuan Hsiao T'sao* (a book treating of equations, right triangles, and the arithmetic of computing rods). Published in the Ching Yuan Yamen, 1898.

Lao Nai Hsuan, *Ch'ou Suan Chien Shih* (Easier course in computing rods), 1897.

Lao Nai Hsuan, *Ku Ch'ou Suan K'ao Shih Hsu Pien* (The old computing rods continued). Published in the Wu Ch'iao Yamen, 1900.

The above works are all in the library of Professor David Eugene Smith.

ON THE MATHEMATICAL SIGNIFICANCE OF THE CHINESE HO T'U AND LO SHU.

By DAVID CHIN-TE CHENG, Hinghwa City, Fukien, China.

The *K'iu-ch'ang Suan-shu*, or Arithmetic in Nine Sections,[1] which probably was written by Li Shou [2] during the reign of Huang ti, the Yellow Emperor, who

[1] Smith, *History of Mathematics*, vol. I, pp. 31–33; Cajori, *History of Mathematics*, p. 71; Smith and Mikami, *History of Japanese Mathematics*, pp. 11–13.

[2] Or Li Shu. See Wu Sheng Chuen, *Kang Chien I Chih Lu* (Briefer History of China), monograph 1, p. 4; Yüan Yüan, *Chou Jen Chuan* (Biography of Chinese Mathematicians), monograph 1, p. 1.

began his reign in the year 2704 B.C.,[3] and the *Chóu-peï Suan-king*,[4] which related to number mysticism, mensuration, and astronomy, in the dialogues between the Prince Chóu-kung and his minister Shang Kao in the Chow dynasty, about 1105 B.C., have been regarded as the oldest books on mathematics in China and perhaps the world. According to tradition, the origin of Chinese mathematics can be traced back to the *Ho-t'u*,[5] a figure appearing on the back of a dragon horse during the reign of Fuh-hi (2852–2738 B.C.[6]). Basing his work upon this figure and after careful observations of the appearance on the heavens and the earth Fuh-hi drew the *Pa-kua*,[7] or the eight trigrams, so familiar in Chinese mysticism. Tradition also states that the *Lo-shu*, the world's oldest specimen of a magic square,[8] was first found upon the back of a tortoise which appeared to the Emperor Yu (*c.*2200 B.C.), when he was embarking on the Yellow River. Hsia Hou Yang (*c.*550) repeats the legend, saying that "Mathematics (Chinese) began with the Emperor Fuh-hi, and Li Shou wrote the *Nine Sections* in the Yellow Emperor's time";[9] and Mo Jo (*c.*1303) refers to the same period and to the two mystic symbols, as follows: "The number 1 is the beginning of all things and from 1, doubling continuously, we have 2, 4, 8, etc. Is this not a natural mathematical process? This process was derived from the *Ho-t'u* and the *Lo-shu*."[10] Mei Wen Ting, the celebrated historian (1693), remarks that "since the appearance of the *Ho-t'u* and the *Lo-shu* we have the odd and even numbers"[11]; and Wu Shih Hsien (1750) says that "Mathematics does not begin with Sun Tze[12]; with the *Ho-t'u* and the *Lo-shu* we have both odd and even numbers."[13]

Of course these traditions are not considered as historical records by Chinese scholars any more than they are by those of the West. They are looked upon simply as part of the poetry of my people, just as stories of King Arthur and his Table Round are looked upon as part of the folklore of England, and as scholars generally look upon the stories of Wilhelm Tell or the tales of Captain John Smith and of Captain Kidd. The *Ho-t'u* and the *Lo-shu* exist, we do not know their origin; but there have grown up, in relation to them, certain mathematical theories which do not seem to have been made known as yet to European and American students, and which it is the purpose of this paper to present.

There are two theories relating the origin of the Chinese mathematics among

[3] Smith, vol. I, p. 24.

[4] Smith, vol. I, pp. 29–31; Cajori, p. 71; Smith and Mikami, p. 9; Mikami, *The Development of Mathematics in China and Japan*, p. 4.

[5] Smith, vol. I, p. 29.

[6] Smith, vol. I, p. 23.

[7] Smith, vol. I, pp. 25–27.

[8] Smith, vol. I, p. 28; Cajori, pp. 76–77.

[9] Hsia Hou Yang, *Hsia Hou Yang Suan Ching*, introduction.

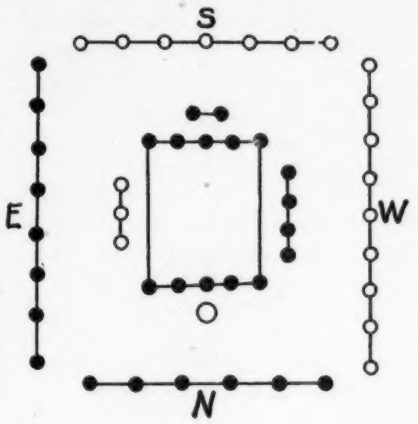
[10] Chu Shih Chieh, *Szu Yuan Yu Chien* (on Chinese algebra). Introduction by Mo Jo.

[11] Mei Wen Ting, Arithmetic. Introduction.

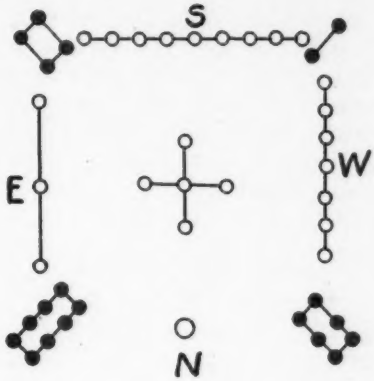
[12] The date of Sun Tze who wrote *Sun Tze Suan Ching* may be about 350 B.C., or possibly as late as the first century. It is very uncertain.

[13] Tan Wen, *Shu Hsueh Hsun Yuan* (To find the Origin of Mathematics). Introduction by Wu Shih Hsien.

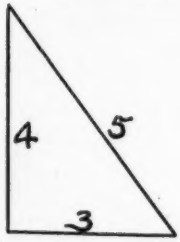
some of the Chinese writers, both of which are based on the figures of the *Ho-t'u* and the *Lo-shu*, and these will now be set forth.



Ho-t'u



Lo-Shu



4	9	2
3	5	7
8	1	6

I. Emperor K'ang Hsi of the Ch'ing dynasty, in 1713 A.D., published two series of his celebrated books containing logarithmic tables. These were issued in fifty-three monographs entitled *Shu Li Ching Yun* (*Principles of Mathematics*) and treating of arithmetic, geometry, and mensuration. In his first monograph of the first series he traced the beginnings of Chinese mathematics. He stated that "addition and subtraction were derived from the *Ho-t'u*, and that multiplication and division were derived from the *Lo-shu*." [14] The explanation is substantially as follows:

a. The origin of addition and subtraction. According to Emperor K'ang Hsi, the numbers of the *Ho t'u* begin with one, the center is five, and they end with ten. The numbers 1, 3, 5, 7, and 9 are odd or masculine numbers, and 2, 4, 6, 8, and 10 are even or feminine numbers.[15] As to the position of the numbers

[14] Emperor K'ang Hsi, *Shu Li Ching Yun*, Chapter I.

[15] In the *Ho t'u* and *Lo shu* the masculine numbers are represented by white (celestial) circles, and the feminine numbers by black (terrestrial) ones.

in the *Ho-t'u*, 1 is the beginning of numbers, it represents the masculine principle and it is placed on the colder north; 2, which represents the feminine principle, is placed on the warmer south; 3, the masculine, is placed east, the position of the rising sun; 4, the feminine, is placed on the west; 5, the masculine, is placed in the center,—the only position left vacant, the other four directions having been taken; 6 is the sum of $5 + 1$, whence $6 - 5 = 1$, and therefore it has the same position as the number 1; 7, masculine, is the sum of $5 + 2$, whence $7 - 5 = 2$, and therefore it has the same position as the number 2; 8, feminine, is the sum of $5 + 3$, whence $8 - 5 = 3$, and therefore it has the same position as the number 3; 9, masculine, is the sum of $5 + 4$, whence $9 - 5 = 4$, and therefore it has the same position as the number 4; 10, feminine, is the sum of $5 + 5$, whence $10 - 5 = 5$, and therefore it has the same position as the number 5. This, according to K'ang Hsi, was the beginning of addition and subtraction in China.

b. The origin of multiplication and division. According to the same writer, the explanation of the positions of the numbers in the *Lo-shu* is as follows: 5 is placed in the center as a person wearing the numeral 9 as his hat; the numbers 3 and 7 are placed on the right and left, the numbers 2 and 4 occupy the position of the shoulders, and the numbers 8 and 6 represent the legs and stand on the number 1. There are five odd or masculine numbers and four even or feminine numbers. Number 1 is the beginning of the numbers, and hence it does not move. The numbers begin with 2 and 3. Beginning with 3 and proceeding clockwise with the odd numbers, we have 3, 9, 7, and 1. Beginning with 2 and proceeding counter-clockwise, we have the following even numbers: 2, 4, 8, and 6. The masculine number 3 is placed on the east, meaning that all things grow from the orient. Beginning at the east and moving to south, we have $3 \times 3 = 9$; moving to the west, we have $3 \times 9 = 27$, and taking away the complete number 20 we have the remainder 7; moving to the north, we have $3 \times 7 = 21$, and taking away the complete number 20 we have the remainder 1. Beginning with the feminine number 2, located in the southwest and representing the right shoulder, and moving to the southeast, we have $2 \times 2 = 4$, which is located on the left shoulder; moving to the northeast we have $4 \times 2 = 8$, which is located on the right leg; and moving to the northwest we have $8 \times 2 = 16$, and taking away the complete number 10 we have the remainder 6, located on the left leg. If we add the numbers horizontally, vertically, or diagonally, the sum is always fifteen, and so we have in the *Lo-shu* what is known as the magic square. The total is 45 which is the product of 9×5. This is therefore asserted to be the beginning of multiplication and division in China.

II. The second theory, held by some of the writers, is that the *Ho-t'u* was the origin of right triangle or the Pythagorean Theorem, and the *Lo-shu* was the origin of addition, subtraction, multiplication, and division. Li Shih Hsiung (1673) stated "the right triangle was derived from the *Ho-t'u*, and addition, subtraction, multiplication, and division were derived from the *Lo-shu;* the whole of the *Nine Sections* was related to the right triangles." [16] Tai Ti Yuan (1750)

[16] Fang Chung T'ung, *Shu Tu Yen*, Introduction by Li Shih Hsiung.

stated that "the *Nine Sections* were derived from the *Ho-t'u* and the *Lo-shu*." [17] Tung Yung (1750) stated that "the *Nine Sections* treated of the right triangle, which in turn was derived from the *Ho-t'u*, and that addition, subtraction, multiplication, and division were derived from the *Lo-shu*. Therefore the *Ho-t'u* and the *Lo-shu* were the origins of mathematics." [18] Tan Wen (1750), in the first chapter of his *Shu Hsueh Hsun Yuan* (*To Find the Origin of Mathematics*), stated that "the theory of numbers began with the *Ho-t'u* and the *Lo-shu*."

According to Tan Wen, Fuh-hi derived the *Pa-kua* from the *Ho-t'u*, and the direction of movement proceeded from north to east, thence to south, thence to the center, thence to west, and finally to the north. In the *Ho-t'u* the sum of the odd numbers 1, 3, 5, 7, and 9 is 25, and the sum of the even numbers 2, 4, 6, 8, and 10 is 30, the total number being 55. Adding 1 and 10 we have 11, and this multiplied by 10 and divided by 2 gives 55. Tan Wen's explanation as to the position and arrangement of numbers in the *Lo-shu* are the same as that given by Emperor K'ang Hsi.

Fang Chung T'ung (1687) in his *Shu Tu Yen*, a book treating of geometry, the abacus, arithmetic, the computation rods, and the *Nine Sections*, asserts that "The *Nine Sections* treats of the right triangle, which in turn was derived from the *Ho-t'u*, and hence the *Ho-t'u* was the origin of mathematics." [19] The numbers in the figure of the *Ho-t'u*, according to Fang Chung T'ung, are based upon the right triangle, having the hypotenuse 5, and the sides 3 and 4. The number 1 is the difference between the hypotenuse 5 and the side 4, or the side 4 minus the side 3; the number 2 is the difference between the hypotenuse 5 and the side 3; the number 3 is the side 3; the number 4 is the side 4; the number 5 is the hypotenuse; the number 6 is the difference between the hypotenuse 5 and the side 4, plus the hypotenuse 5; the number 7 is the difference between the hypotenuse 5 and the side 3, plus the hypotenuse 5, or it is the sum of sides 4 and 3; the number 8 is the sum of the hypotenuse 5 and the side 3; the number 9 is the sum of the hypotenuse 5 and the side 4. The *Chóu-peï Suan-king* treats of the right triangle having the sides 3 and 4 and the hypotenuse 5, 25 being the square of the hypotenuse. The sum of the numbers in the *Ho-t'u* is 55; the middle number 5 is of no consequence, and 50 is the sum of the squares of 3, 4, and 5. The number 3, which is masculine and is located at the left, added to the hypotenuse 5, equals 8, and so the numbers 3 and 8 have the same position at the left of the figure (east); the number 4, which is feminine and at the right, added to the hypotenuse 5, equals 9, and so the numbers 9 and 4 have the same position at the right (west); and the number 5, which is masculine and is the hypotenuse, is equal to $\sqrt{3^2 + 4^2}$ and stands in the center. The hypotenuse 5 minus the side 3 equals 2, which is located above; and the hypotenuse 5 minus the side 4 equals 1, which is located below. The hypotenuse 5 minus the side 3

[17] Tan Wen, *Shu Hsueh Hsun Yuan* (*To Find the Origin of Mathematics*). Introduction by Tai Ti Yuan.

[18] Tan Wen, *Shu Hsueh Hsun Yuan* (*To Find the Origin of Mathematics*). Introduction by Tung Yung.

[19] Fang Chung T'ung, *Shu Tu Yen*, Chapter I.

equals 2, and this plus the hypotenuse 5 equals 7; therefore the numbers 7 and 2 have the same position; the hypotenuse 5 minus the side 4 equals 1, and this plus hypotenuse 5 equals 6; therefore the numbers 6 and 1 have the same position. This is the origin of the right triangle.

As to the *Lo-shu*, Fang T'ung stated that "addition, subtraction, multiplication, and division are all derived from the *Lo-shu*," and that "the addition of a masculine number and a feminine number gives a masculine number";[20] for example, $1 + 6 = 7$, $7 + 2 = 9$, $9 + 4 = 13$, and taking away the complete number 10 we have 3; and $3 + 8 = 11$, from which, by taking away the complete number 10, we have 1. Numbers begin with the masculine ones, and these govern the feminine ones. This was the beginning of addition. Subtracting masculine numbers from feminine ones we have $6 - 1 = 5$, $8 - 3 = 5$; and subtracting feminine numbers from masculine ones we have $9 - 4 = 5$, $7 - 2 = 5$. Taking away the side numbers, and retaining the middle number 5, was the beginning of subtraction. In the *Lo-shu* the opposite numbers 1 and 9 have the following relations: $9 \times 1 = 9$, $1 \times 9 = 9$, $9 \div 1 = 9$, $9 \div 9 = 1$; the opposite numbers 2 and 8 give $2 \times 8 = 16$, $8 \times 2 = 16$, $16 \div 8 = 2$, and $16 \div 2 = 8$; the opposite numbers 3 and 7 give $3 \times 7 = 21$, $7 \times 3 = 21$, $21 \div 3 = 7$, and $21 \div 7 = 3$; the opposite numbers 4 and 6 give $4 \times 6 = 24$, $6 \times 4 = 24$, $24 \div 4 = 6$, and $24 \div 6 = 4$. This was the beginning of multiplication and division.

Such are some of the fanciful theories concerning the significance of the two most ancient pieces of Chinese mathematics that we have. They show the play of fancy and imagination in the work of the scholars of my country in the period before the modern introduction of European science. They lack the mathematical insight of the West, but the West lacks the poetic insight of the East. If one were to be deprived of either, with which would it be better for him to dispense? Perhaps the best answer is to say that it is not necessary to lack both. If so, which will teach the other this fact?

QUESTIONS AND DISCUSSIONS.

EDITED BY C. F. GUMMER, Queen's University, Kingston, Ont., Canada.

The department of Questions and Discussions in the MONTHLY is open to all forms of activity in collegiate mathematics, including the teaching of mathematics, except for specific problems, especially new problems, which are reserved for the separate department of Problems and Solutions.

REPRINT OF OLD QUESTIONS.

It is thought that some of the questions of this department which have hitherto remained unanswered should be reprinted for the attention of new readers and others who had previously overlooked them.

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[20] Fang Chung T'ung, *Shu Tu Yen*, Chapter I.

34 [*1917*, 134, 341; *1920*, 114, 301, 405, 460; *1921*, 19, 125; *1922*, 159]. Given the mixed integral and functional equation

$$\int_{x=0}^{x=h} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental value as it has to do with the most general solid whose volume is given by the prismatoid formula.[1]

Previous contributors have shown that $f(x)$ may have *one*, but not *six*, continuous derivatives.

39 [*1900*, 256; *1921*, 125; *1922*, 160].[2] There are certain problems in geometry which are simple in statement but can be reduced only to very complicated problems in transcendental analysis. Following are several examples of the type of problem in question.

1. What is the smallest plane area within which a given figure can be turned through a complete revolution? It is not implied that the figure should revolve about a fixed point, but merely that in the course of its motion it should have every possible orientation in the plane. The problem may be modified by considering only convex areas.

An interesting special case is that in which the given figure is a segment of a straight line.[3] In this case it has been conjectured by Professors Osgood and Kubota that the smallest area may be bounded by a three-cusped hypocycloid; if we consider only convex areas, perhaps the result will be an equilateral triangle. I have no indication of a proof.

2. For every closed convex curve of area P there is an n-sided circumscribed polygon of least area Q and an inscribed polygon of greatest area R. For a fixed value of the integer n and for all convex curves, what is the upper limit of Q/P and what is the lower limit of R/P? I have succeeded only in proving that for the case $n = 3$ the upper limit of Q/P is 2.

3. Let the area of a simple closed curve A be a. Remove from A the greatest possible area a_1 similar to another closed curve B. From the remaining figure remove the greatest possible area a_2 similar to B. Continue this process indefinitely. Is it or is it not true that

$$a_1 + a_2 + a_3 + \cdots = a?$$

I have proved the statement to be true in the special case where A is convex and B is a circle.

4. Let a given closed convex curve K have the property that a given triangle whose angles are incommensurable with π can be revolved completely within K (see part 1 of this question), always remaining inscribed to K. What may the curve K be? Can any other curve except a circle satisfy the conditions?

43 [*1921*, 260; *1922*, 161]. Is any rapid method known for the evaluation of the Sylvester determinant met with so often in elimination by the dialytic method? It would seem that there must be, both on account of its interesting shape and of its frequent occurrence.

45 [*1921*, 305; *1922*, 161].

See the reply appearing in this number, which disposes of one part of the question, but leaves something to be done with regard to the second part.

46 [*1922*, 210]. A geometrical construction will often become impracticable in special cases where some of the construction points are imaginary, although the final result is real. Is there any system according to which a construction failing in this way may be replaced by one that will work?

For example, let a line l_1 cut a circle c_1 in P_1 and Q_1, and let a line l_2 cut a circle c_2 in P_2 and Q_2. If the four points P_1, Q_1, P_2, Q_2 are real, the intersections of P_1P_2 with Q_1Q_2 and P_1Q_2 with P_2Q_1 may be found directly. If the four points are imaginary, the intersections named will still

[1] A summary of results so far obtained in this problem is given in the last reprint of questions (*1922*, 159). These results leave the following question to be answered:

How many continuous derivatives may a real function $f(x)$ (not a cubic or lower polynomial) possess while satisfying the given equation for every h in a certain real interval $0 < h < H$?

[2] As contributed by Professor S. Kakeya.

[3] For a varied form of this problem see W. B. Ford, "On Kakeya's Minimum Problem," *Bulletin of the Amer. Math. Soc.*, vol. 28, 1922, pp. 45–53. See also Question **42** in this department (*1921*, 65, 126; *1922*, 161).—EDITOR.

be real, and there ought to be a simple way of getting them as the points common to a line and a circle.[1]

51 [*1924*, 85]. Can any reader supply approximate formulas for the problem of a cable suspended from two points at different levels?

NEW QUESTION.

55. Is it possible by ruler and compasses to construct an angle equal to one radian?[2]

REPLIES TO QUESTIONS.

45 [*1921*, 305; *1922*, 161]. Is every non-trivial solution in integers of the equation $t^3 = x^3 + y^3 + 1$ expressible in the form $x = 9r^4 - 3r$, $y = 9r^3 - 1$, $t = 9r^4$? If there are non-trivial solutions not expressible in this form, can a general solution be found?

REPLY BY N. B. MITRA, Ewing Christian College, Allahabad, India.

The answer to the first part of the question is in the negative, as the following investigation will show.[3]

The problem is to solve in integers the equation

$$x^3 + y^3 + z^3 = 1. \tag{1}$$

We have

$$x^3 + y^3 = 1 - z^3. \tag{2}$$

If $x + y = 1 - z$, then from (2), $x^2 - xy + y^2 = 1 + z + z^2$. Eliminating z we get $xy - (x + y) + 1 = 0$ which gives the trivial solutions, $x = 1$, $y = -z$ and $y = 1$, $x = -z$. If $x + y \neq 1 - z$, then integers p and q, prime to each other (one of them may be unity), exist such that

$$\frac{1 - z}{x + y} = \frac{p}{q}. \tag{3}$$

We may without loss of generality suppose p to be positive and this we do. We shall show (see below) that in this case q cannot be negative.

From (2) and (3),

$$q(x^2 - xy + y^2) = p(1 + z + z^2). \tag{4}$$

Eliminating z from (3) and (4),

$$x^2(p^3 - q^3) + xy(2p^3 + q^3) + y^2(p^3 - q^3) - 3p^2q(x + y) + 3pq^2 = 0. \tag{5}$$

In order that integral solutions of (1) other than the two trivial ones found above may exist, equation (5) must be capable of affording integral values of x and y.

[1] Perhaps it would be better to omit the last ten words of the question. It should be possible for some reader to solve at least the particular problem here proposed. The more general question, though probably admitting an affirmative answer, calls for a well-planned representation of imaginary points.

[2] Proposed in the course of Discussion I appearing below.

[3] The solution given by Professor Bradley (*1921*, 307) was arrived at by me by quite a different method and was published in the *Journal of the Indian Mathematical Society*, vol. XIII (Feb. 1921), p. 17.

Since p and q are prime to each other, equation (4) cannot subsist unless p is a factor of $x^2 - xy + y^2$ and q of $1 + z + z^2$. Now $x^2 - xy + y^2$ and $1 + z + z^2$ are both of the form $a^2 + 3b^2$, and factors of numbers of this form are always of this form also. Hence p and q must both be of the same form. In other words p and q are both of the form $3^m f^2 N$, where $m = 0$ or 1, f is any integer and N is composed entirely of factors which are primes (including 1) of the form

$$6n + 1. \tag{6}$$

By giving any suitable values to p and q consistent with (6) we can get all the possible integral solutions of (5) or else satisfy ourselves that there are none.

Equation (5) may be thrown into the form (always, since $p \neq q$)

$$X^2 - DY^2 = H, \tag{7}$$

where

$$X = 3q^2\{x(4p^3 - q^3) - 3p^2q\}, \tag{8.1}$$

$$Y = (2p^3 + q^3)x + 2(p^3 - q^3)y - 3p^2q, \tag{8.2}$$

$$D = 3q(4p^3 - q^3), \tag{8.3}$$

$$H = 36pq^3(p^3 - q^3)^2. \tag{8.4}$$

Equations (8.1) and (8.2) show that X and Y are integral. Hence equation (7) must be capable of affording integral values of X and Y, if (1) is possible.

If q is negative, D and H are both negative and (7) has no real solutions. Hence for the possibility of (1), q must be positive.

If $q > p\sqrt[3]{4}$, D is negative and H is positive; therefore the number of integral solutions of (7) in this case is limited and there may be none.

If $q < p\sqrt[3]{4}$, D and H are both positive; in this case, if D is not a perfect square, (7) will admit of an infinite number of solutions, provided H, if $< \sqrt{D}$, occurs among the divisors belonging to the development of $\sqrt{D}$ as a S.C.F. and all the solutions may be obtained in the usual way. For other values of H there are either no solutions or an infinite number. If however D is a perfect square, the number of solutions is limited and there may be none.

From the values of X and Y thus obtained we are to pick out those which give integral values of x and y from (8.1) and (8.2); and from the values of x and y we are to reject those which do not give integral values of z from (3). In the particular case $q = 1$, every integral value of x and y gives an integral value of z.

The values of X and Y obtained as above form a periodic series to any given modulus; thus for any given values of p and q, a finite number of trials will determine whether they give integral values of x and y and of z.

The following numerical examples will illustrate the method.

(*a*) Take $p = 3$, $q = 1$. Then we get $X^2 - 321Y^2 = 3 \cdot 156^2$, where $X = 321x - 81$ and $Y = 55x + 52y - 27$.

Putting $X = \pm 156X_1$, $Y = \pm 156Y_1$, we get $X_1^2 - 321Y_1^2 = 3$, which gives $X_1 = 18r \pm 321s$, $Y_1 = 18s \pm r$, where $r^2 - 321s^2 = 1$, so that

$$r = \frac{1}{2}\{(215 + 12\sqrt{321})^n + (215 - 12\sqrt{321})^n\},$$

$$s = \frac{1}{2\sqrt{321}}\{(215 + 12\sqrt{321})^n - (215 - 12\sqrt{321})^n\},$$

where n is any integer. Putting $n = 0$ and 1, we get $r = 1$ and 215, $s = 0$ and 12; $X_1 = 18$ and 7722, $Y_1 = 1$ and 431: whence $x = 9$ and 3753, $y = -6, -12$ and $-2676, -5262$, and $z = -8, 10$ and $-3230, 4528$.

Hence we get the following solutions:

$$-6^3 + 9^3 - 8^3 = 1, \qquad (a, 1)$$

$$-12^3 + 9^3 + 10^3 = 1, \qquad (a, 2)$$

$$-2676^3 + 3753^3 - 3230^3 = 1, \qquad (a, 3)$$

$$-5262^3 + 3753^3 + 4528^3 = 1. \qquad (a, 4)$$

Giving other values to n, we may get as many solutions as we like.

Again, putting $X = \pm 156X_1/f$, $Y = \pm 156Y_1/f$ and solving the equation $X_1^2 - 321Y_1^2 = 3f^2$, where f is any integral factor of 156, we may get as many other solutions as we like.

(*b*) Take $p = 4$, $q = 1$. Then we get $X^2 - 85Y^2 = 84^2$, where

$$X = 85x - 16, \qquad Y = 43x + 42y - 16.$$

All the integral solutions of $X^2 - 85Y^2 = 84^2$ are obtained by putting $X = \pm 84r/f$, $Y = \pm 84s/f$ and solving in integers $r^2 - 85s^2 = f^2$, where f is any factor of 84.

Thus, taking $f = 1$, we have $X = \pm 84r$, $Y = \pm 84s$, $r^2 - 85s^2 = 1$. Hence in this case,

$$r = \frac{1}{2}\{(285769 + 30996\sqrt{85})^n + (285769 - 30996\sqrt{85})^n\},$$

$$s = \frac{1}{2\sqrt{85}}\{(285769 + 30996\sqrt{85})^n - (285769 - 30996\sqrt{85})^n\}, \qquad (9)$$

where n is any integer.

The residues, modulo 85, of the successive values of r form a periodic series of which the period is (84, 1). None of these make x integral.

Taking $f = 2$, we again find no integral values for x.

Taking $f = 3$, we have $X = \pm 28r_2$, $Y = \pm 28s_2$, where $r_2^2 - 85s_2^2 = 9$. Hence we get $r_2 = 37r \pm 85 \cdot 4s$; $s_2 = 37s \pm 4r$, r and s having the values given by (9). Here we get $X = -12r \pm 112s - 16(r - 1)/85$ or $12r \pm 112s + 16(r + 1)/85$. The former of these make X integral if $r \equiv 1 \pmod{85}$ and the latter if $r \equiv 84 \pmod{85}$; and this is the case according as n is even or odd in (9).

Thus, taking $n = 0$, we get the solution

$$-12^3 + 10^3 + 9^3 = 1, \qquad (b, 1)$$

and, taking $n = 1$, the two solutions

$$11468^3 - 14258^3 + 11161^3 = 1, \qquad (b, 2)$$

$$6954572^3 - 5593538^3 - 5444135^3 = 1. \qquad (b, 3)$$

If we take $f = 7$, we get the solutions

$$172^3 - 138^3 - 135^3 = 1, \qquad (b, 4)$$

$$-812^3 + 1010^3 - 791^3 = 1, \qquad (b, 5)$$

$$464196268^3 - 577145658^3 + 451797561^3 = 1, \qquad (b, 6)$$

$$-98196140^3 + 78978818^3 + 76869289^3 = 1. \qquad (b, 7)$$

Putting $f = 14$, we obtain

$$67402^3 - 83802^3 + 65601^3 = 1. \qquad (b, 8)$$

And so on.

Equation (5) may also be thrown into the form

$$(p^3 - q^3)u^2 + (2p^3 + q^3)uv + (p^3 - q^3)v^2 = m,$$

where

$$u = (4p^3 - q^3)x - 3p^2q, \qquad v = (4p^3 - q^3)y - 3p^2q, \qquad m = 3pq^2(4p^3 - q^3)(2q^3 - p^3).$$

Thus the problem of obtaining integral solutions of (5) is identical with that of the representation of the integer $2m$ in the binary quadratic form $(2p^3 - 2q^3, 2p^3 + q^3, 2p^3 - 2q^3)(u, v)$.

Of the values of u and v thus found we are to pick out those which satisfy $u \equiv v \equiv -3p^2q \pmod{4p^3 - q^3}$. From these values of x and y we are to choose those which make z integral.

If $u = v$, then $x = y$, and (2) reduces to $1 - z^3 = 2x^3$, which has no other integral solution except the trivial ones $x = 0, z = 1$ and $x = 1, z = -1$.

N.B. The six results $(a, 1)$, $(a, 2)$, $(b, 4)$, $(b, 5)$, $(b, 2)$ and $(b, 8)$ were given by the late Mr. Ramanujan in the form of a question (without demonstration) in the *Journal of the Indian Mathematical Society*, vol. VII (1915), Question No. 681.

I. The Definition of Radian.

By A. A. Bennett, Lehigh University.

It is customary to define the word "radian," as used in the circular measure of angles, by some such statement as the following: "A radian is the angle subtended at the center of a circle by an arc equal in length to the radius." Probably few teachers of trigonometry would care to be quizzed on the exact meaning that they wish to convey by the words, "an arc equal in length to the radius." This is a notoriously deep hole which most of us are relieved to find the students unquestioningly swimming over without attempting to touch bottom. It would seem inappropriate with only such a harmless application in mind to make vague mention of the difficulties in the limit notions of real variables. Were this the only possible method of definition, the current practice could only deserve regretful condonement, but this is not the case. Of course essentially the same

situation has arisen in the previous training of the student where the notion of π is first introduced. It is this fact that makes it possible to avoid the difficulty here. The student can be made to understand that the symbol π is used to denote a certain definite real number approximately equal to $3\frac{1}{7}$. This number, π, is definable by series in many ways and does not require for its definition any comparison of lengths of arc. As a consequence of various theorems involving limiting processes, it is *convenient* to *define* the ratio of the circumference to the diameter as a number and in particular as this number, π. Throughout trigonometry it must be assumed that for each positive number, a, there is a unique angle, which is the $(1/a)$th part of a straight angle. The "construction" may be regarded as involving at worst the limit of an infinite sequence of bisections completely determined by the number, a. This is part of the continuity assumed in the topic and is not subject to ruler and compasses restrictions such as appear in the question of the trisection of an angle. One might remark, parenthetically, that even the angle of one degree cannot be constructed with ruler and compasses. In particular the angle which is the $(1/\pi)$th part of a straight angle is *defined* as a radian. In the same sense that one may assume that the diameter goes into the circumference exactly π times, one may say that a central angle of one radian subtends an arc equal to the radius. The same difficulties attend both statements, and neither is logically necessary in trigonometry, although the origin of the term "radian" is not without interest.

The title of this Department suggests that a question not unrelated to the brief discussion made above may be appropriate here. The question I here propose will probably remain long unanswered. "Is it possible by ruler and compasses to construct an angle equal to one radian?" Presumably not, but it is not clear that the transcendence of π implies this conclusion.

II. Note on the Continuity of a Function Defined by a Definite Lebesgue Integral.

By H. J. Ettlinger, University of Texas.

In the recent June-July number of the Monthly, R. L. Jeffery proved the following theorem.[1]

Hypothesis: 1. *$f(x, y)$ is a real bounded function of (x, y) on the square $a \leq x \leq b$, $a \leq y \leq b$.*

2. *$f(x, y)$ is a summable function of x on (a, b) for each y in (a, b).*

3. *$f(x, y)$ is a continuous function of y at $\bar{y}$, for every x in (a, b), except a null set M_0.*

Conclusion: $F(y) = L\int_a^b f(x, y)dx$ *is continuous at every point $\bar{y}$ in (a, b).*

The proof of this theorem may be obtained as a direct result of the Duhamel-Moore theorem as follows.[2]

[1] The continuity of a function defined by a definite integral, this Monthly (*1925*, 297–299).

[2] R. L. Moore, On Duhamel's theorem, *Annals of Mathematics*, second series, vol. 16 (1911), pp. 45–49. See also H. J. Ettlinger, A simple form of Duhamel's theorem and some new applications, this Monthly (*1922*, 241).

Let $y_1, y_2, \cdots, y_n, \cdots$ be any infinite sequence of values in (a, b) such that $\lim_{n\to\infty} y_n = \bar{y}$. Then for any fixed x in (a, b) not in M_0, $\lim_{n\to\infty} f(x, y_n) = f(x, \bar{y})$. Consider $|F(\bar{y}) - F(y_n)| = \left| L\int_a^b [f(x, \bar{y}) - f(x, y_n)]dx \right| \leq \sum_{i=1}^{n} h_{in}e_{in}$, where, following Lebesgue's definition of integrability, h_{in} represents the points of division of the range of the function $|f(x, \bar{y}) - f(x, y_n)|$, and e_{in} is the measure of the set of points $E[h_{i-1,\,n} \leq |f(x, \bar{y}) - f(x, y_n)| < h_{i,\,n}]$.[1] Now h_{in} is bounded for all values of i ($\leq n$) and n, since $f(x, \bar{y})$ and $f(x, y_n)$ are bounded for all values of (x, y) in the square. If $\bar{x}$ is not in M_0, $\lim_{n\to\infty} h_{i_{P_n}n} = 0$, where P is the point in (a, b) corresponding to $x = \bar{x}$. Hence by the Duhamel-Moore theorem, $\lim_{n\to\infty} \sum_{i=1}^{n} h_{in}e_{in} = 0$. Hence $\lim_{n\to\infty} F(y_n) = F(\bar{y})$, *i.e.*, $F(y)$ is continuous at $y = \bar{y}$.

The above process is the equivalent of proving a generalization of a theorem due to Lebesgue[2] which may be stated as follows: *If a bounded sequence of measurable functions $f_n(x)$ converges for each x in (a, b) except a null set M_0 to a limit function $f(x)$, then $\lim_{n\to\infty} L\int_a^b f_n(x)dx = L\int_a^b f(x)dx$.*

It can be readily shown that if hypothesis 3 be modified to require integrability in y, for every x in (a, b), except a null set M_0, then $F(y)$ is an integrable function of y on (a, b).

RECENT PUBLICATIONS.

EDITED BY W. B. CARVER, Cornell University, to whom books and communications should be sent.

REVIEWS.

History of Mathematics. By DAVID EUGENE SMITH. Ginn and Company, 1923, 1925. Volume I, *General Survey of the History of Elementary Mathematics.* xxii + 596 pages. Price $4.00. Volume II, *Special Topics of Elementary Mathematics.* xii + 725 pages. Price $4.40.

The excellence of this history of elementary mathematics is due primarily to the many years which Professor Smith has devoted to a first hand study of original sources. He had long collected early and rare books not only for himself, but more extensively for Mr. George A. Plimpton, also of New York, whose remarkable library contains the most important and extensive collection of early arithmetics in the world, besides many rare books and manuscripts on other fields of elementary mathematics. The constant enthusiastic study of these and other sources during so many years has made Smith an outstanding authority on the history of elementary mathematics. Fortunately he writes in a very clear and pleasing style. In these two volumes he has presented an authoritative and

[1] *Leçons sur l'intégration*, Gauthier-Villars, Paris (1904), p. 112.
[2] *L. c.*, p. 114.

entertaining history of elementary mathematics, which should be accessible to every one who teaches or is otherwise interested in mathematics.

> This work has been written for the purpose of supplying teachers and students with a usable text book on the history of elementary mathematics, that is, of mathematics through the first steps in the calculus. . . . The general plan adopted in the preparation of this work is that of presenting the subject from two distinct standpoints, the first, as in Volume I, leading to a survey of the growth of mathematics by chronological periods, with due consideration to racial achievements; and the second, as in Volume II, leading to a discussion of the evolution of certain important topics. . . . A general historical presentation is desirable for the purpose of relating the development of mathematics to the development of the race, of revealing the science as a great stream rather than a static mass, and of emphasizing the human element, but . . . this ought to lead to a topical presentation by which the student may understand something of the life history of the special subject which he may be studying. (*Quoted from author's preface.*)

Each chapter of Volume I treats a certain period; these periods are separated by the following dates: 1000 B.C., 300 B.C., 500 A.D., 1000, 1500, 1600, 1700. The subdivision of a chapter is usually by countries. For such a subdivision there is discussed in chronological order the contributions made by influential mathematicians of a given country in a given period.

Volume II treats the history of arithmetic, geometry, algebra, trigonometry, and calculus. The 68 pages of Chapter VII deal with mathematical recreations, history of commercial problems, puzzle problems leading to indeterminate equations in integers, and magic squares. The 42 pages of Chapter IX treat the history of measures of weight, length, metric system, and time (calendars and timepieces).

An important feature throughout is the abundance of cuts, diagrams, maps, reproductions of engravings, paintings, lithographs, and photographs, as well as facsimiles of pages of rare books and manuscripts.

What Smith has done so ably for elementary mathematics needs to be done for the various fields of advanced mathematics. A dozen men of promise in America should be encouraged to fit themselves to become our future historians of advanced mathematics.

L. E. Dickson.

The History of Mathematics in Europe. By J. W. N. Sullivan. London, Oxford University Press, 1925. 110 pages. Price $1.00.

To outline the history of mathematics within the narrow compass of about a hundred pages, and to do this in a manner attractive to the general reader, is a difficult task which Sullivan has accomplished with remarkable success. The booklet contains pictures of fourteen mathematicians, Nicolas of Cusa, Pacioli, Cardan, Galileo, Napier, Kepler, Descartes, Fermat, Pascal, Wallis, Huygens, Newton, Laplace, Lagrange, and reproductions of portions of two medieval manuscripts. The history is carried down to the close of the eighteenth century. The human interest is enhanced by the insertion of biographical detail. Of Stifel we read,

> His conversion [to Protestantism] was brought about by his discovery that the number 666, the number of the beast in the Book of Revelation, really referred to Pope Leo X, ... If Leo

DeCIMVs be written, the letters MDCLVI make up the number 1656 . . . the name should be followed by the symbol X. The result is now too great by 1000. But the 1000 springs from the letter M which, in this case, must stand for *Mysterium* and not for a number. Thus the required result is obtained.

Doubtless the conversion was well under way before Stifel gave his interpretation of the significance of 666.

In some cases the author missed opportunities to point out the evolution of ideas or notations, as when he failed to connect Descartes' exponential notation with the earlier notations of Chuquet, Bombelli, Stevin, Hérigone and Hume. In a few instances the author overlooked the latest researches, as when he spoke of Egyptian geometry as chiefly connected with land-surveying problems, and made no reference to the remarkable approximation to the area of a circle found in the Ahmes papyrus, nor to the exact computation of the volume of the frustrum of a quadrangular pyramid recently found in the Egyptian "Moscow papyrus." An imperfect characterization of Descartes' great work, *La Géométrie*, lies in the statement, "The wholly new contribution made by Descartes was in importing the idea of motion into geometry." In reality the important new contribution was the geometric interpretation of an algebraic equation containing two variable coördinates. There are no facts to support Sullivan's statement that Napier "had a precursor in Joost Bürgi." Other minor defects, here and there, might be pointed out, but they do not seriously lessen the value of the work as a whole. Students desiring a rapid general survey of the growth of mathematics and caring more for readability than for extreme accuracy of detail will peruse Sullivan's book with pleasure and profit.

FLORIAN CAJORI.

An Introductory Account of Certain Modern Ideas and Methods in Plane Analytic Geometry. By CHARLOTTE ANGAS SCOTT. Second edition with notes and corrections. New York, G. E. Stechert & Co., 1924. xx + 288 pages. Price $4.00.

The first edition of this book was published in 1894 and immediately met with hearty approval, as it furnished about the only account in English of many of the "modern ideas and methods in plane analytic geometry." But the book has long been out of print, and of the recent works which have appeared in the meantime, none in English occupies itself with just this field. In this interval some of the fundamental concepts have undergone extensive changes, and now some of the aspects are rather old-fashioned, but the student of geometry will still find all of it profitable reading, and some of the chapters are as useful today as they ever were. The only change in the new edition is the addition of six pages of notes and corrections, most of which are simpler proofs than those given in the earlier text.

We welcome the reappearance of this instructive and interesting volume.

VIRGIL SNYDER.

North Star Navigation. By L. M. BERKELEY. New York, White Book and Supply Co., 1924. 86 pages. Price $3.75.

This little pamphlet gives what is probably the simplest, most practical and valuable method yet suggested for finding the observer's position on the earth, at the same time giving as a valuable by-product the azimuth of Polaris. It seems strange that so simple a method should not have been developed soon after the invention of reasonably accurate chronometers. High praise is due the author for the elegance and simplicity with which he has presented the method and especially for the clearness of presentation of the mathematical theory and the adaptation to the practical needs of the navigator.

It seems to the reviewer that the method of obtaining a ship's position, which is so ably and clearly presented by Mr. Berkeley, can hardly fail to replace the Sumner or St. Hilaire methods, as those have largely displaced the older methods. One of the strongest points in favor of Mr. Berkeley's method is that the position may be found without any knowledge of the approximate position by dead reckoning or even without a knowledge of change of position by dead reckoning.

The only criticism which should be offered is on the notation and the use of terms. Where a notation has been long in current use and so become "classic," it is much better to use the "classic" notation than to introduce a new notation. Also, it is unfortunate to confuse meridians and hour circles as is done on page 26, where, in the first paragraph, the meridians of Greenwich and the north star are spoken of where the hour circles through the zenith of Greenwich and through the north star are intended.

It may further be remarked that the methods developed by Mr. Berkeley are useful to the surveyor in getting azimuth and latitude and will no doubt be taught in many of our engineering colleges provided the tables are made easily available.

S. L. BOOTHROYD.

Στοιχεῖα Ἀναλυτικῆς Γεωμετρίας: μερος πρωτον. By NEILOU SAKELLARIOU, with an introduction by CONSTANTINE CARATHÉODORY. Athens, Press of E. and I. Mplazoudaky, 1924. 288 + x pages.

It's no fun being an heir of all the ages. If you are one and write a book, your readers may expect too much of you. Not every Greek mathematician can be a Euclid, an Apollonius or a Carathéodory, and Professor Sakellariou has not set out to produce a κτῆμα ἐς ἀεί, but only to write an introduction to analytic geometry suited to the needs of students in the University of Athens and other institutions of higher learning, and he has, *exceptis excipiendis*, succeeded admirably in this first part, to which Professor Carathéodory has contributed an introduction.

The recent writer in the *Times Literary Supplement*, who rather sadly acquiesces in the Americanization of the world, could find further evidence of it were he to examine this book, for it would remind him, in subject matter and lists of problems, not of Salmon or Clebsch but of a score of American text-books for

beginners. In arrangement, the book differs from most American text-books in that solid analytic geometry makes its appearance very soon; the second chapter treats of planes and straight lines in space. It is not until the fourth that the circle is taken up along with the sphere, while the other conic sections have the final fifth chapter reserved for them.

A fuller treatment of directed lines than American books of similar character usually contain is a praiseworthy feature of the work. The proofs of the theorems of Menelaus and Ceva appear very soon and this adds interest to the earlier part of the book, the most difficult and least interesting part of analytic geometry. Furthermore a list of formulæ at the end of the volume is very convenient for reference. But however excellent a book may be in other particulars, we have a right to complain of the author who does not treat clearly and correctly the fundamental questions of tangents and asymptotes. We can forgive seventy times seven divisions by zero in an elementary book; we may even without asperity allow an author to give a definition of circles which applies to the real case only and then say that every equation of the form

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

represents a circle, but the unpardonable sin is to mislead the student on the meaning of tangent, for that is fundamental, and a Greek should set us a good example in this matter, since his fathers have eaten fewer sour grapes than our own mathematical progenitors.

We can be sure that it was of certain passages on tangents in mathematical works that the Red Queen was thinking when she shook her head and said: "You may call it nonsense if you like, but *I've* heard nonsense in comparison with which this would be as sensible as a dictionary." Euclid's definition of a tangent to a circle as a line which cuts it in only one point is perfectly explicit and, with certain obvious exceptions, is valid for all conic sections and determines the same straight line as the more modern definition of tangent. It is the basis of the so-called discriminant method and its greatest disadvantage is that by its use the opportunity of introducing the fundamental concept of limit is lost. But Professor Sakellariou's definition is neither fish nor flesh for he says: "If we consider a right line, which cuts a curve in two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ and itself turns about the point P_1 in such a manner that P_2 continually approaches P_1 until finally P_2 coincides with P_1, we say that the line is tangent to the curve under consideration at P_1." These words miss the point in each definition of tangent and end in saying—nothing at all. If P_2 is to coincide with P_1, we have only one point on the line and one point will not determine a straight line, whether we call it two points or not. If Euclid's definition is the one meant, the point P_2 has nothing to do with the case and the essential fact that the line shall cut the curve at one point only is omitted. Nor does this definition carry more than a hint of the limit definition of tangent, as the slope of the tangent, under the limit definition of tangent, depends on the slope of the secant when P_2 does NOT coincide

with P_1 and what happens when P_2 coincides with P_1 has nothing to do with it.

Such a definition would be expected to lead our author into trouble and it does so straightway. It is true that if P_1 and P_2 are two *different* points on the circle $x^2 + y^2 = r$, not at opposite ends of a diameter,

$$\frac{y_2 - y_1}{x_2 - x_1} = -\frac{x_2 + x_1}{y_2 + y_1}$$

and that the latter ratio is the slope of the line P_1P_2, but when P_2 coincides with P_1, Professor Sakellariou must not claim that this ratio is still the slope of the line P_1P_2, only one point of which is fixed.

We have laid emphasis on this matter, not because Professor Sakellariou is an uncommon sinner, but precisely for the opposite reason; his treatment of tangents is only too typical, and it must be set down to his credit that his treatment of asymptotes is far superior to that of many text-books with which we are familiar.

W. L. G. WILLIAMS.

ARTICLES IN CURRENT PERIODICALS.

The lists appearing regularly in the MONTHLY of articles in current periodicals are intended to include (1) titles of papers in all mathematical journals published in the United States; (2) titles of mathematical papers and reports published by the national and state academies of science and in journals devoted to general science; (3) titles of mathematical papers by American authors published in foreign journals.

BULLETIN DE LA SOCIETÉ MATHÉMATIQUE DE FRANCE, volume 52, nos. 3–4: "Un problème de probabilités dénombrables" by Norbert Weiner, 569–577; "Sur les séries de Fourier restreintes et la convergence presque partout" by W. H. Young, 578–584.

BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, volume 31, no. 7, July, 1925: "The number of even and odd absolute permutations of n letters" by J. M. Thomas, 303; "The absolute value of the product of two matrices" by J. H. Wedderburn, 304–308; "On the number of representations of an integer as the sum or difference of two cubes" by E. T. Bell, 309–311; "Contact curves of the rational plane cubic" by L. W. Griffiths, 312–317; "Note on the projective geometry of paths" by T. Y. Thomas, 318–322; "The tensor character of the generalized Kronecker symbol" by F. D. Murnaghan, 323–329; "Two general functional equations" by W. H. Wilson, 330–334; "Functional invariants, with continuity of order p, of one parameter Friedholm and Volterra transformation groups" by A. D. Michal, 335–345; "On the distribution of quadratic and higher residues" by H. S. Vandiver, 346–350.

JOURNAL OF MATHEMATICS AND PHYSICS, M. I. T., volume 4, no. 4, July, 1925: "Note on quasi-analytic functions" by Norbert Wiener, 193–199; "File multiplication of ordered determinants" by Lepine Hall Rice, 200–204; "A theory of ordered determinants with application to polyadics" by Frank L. Hitchcock, 205–237; "A new method in the theory of quantics" by Frank L. Hitchcock, 238–256.

MESSENGER OF MATHEMATICS, volume 54, no. 12, April, 1925: "The x functions of Glaisher and class numbers" by E. T. Bell, 186–188.

SCIENCE, volume 62, nos. 1595–1598, July 24 and 31 and August 7 and 14, 1925: "Some mathematical aspects of cosmology" by W. D. MacMillan, 63–72, 96–98, 121–126; "Relation of the restricted to the general theory of relativity" by W. F. G. Swann, 145–147.

THE QUARTERLY JOURNAL OF PURE AND APPLIED MATHEMATICS, volume 50, no. 2: "On surfaces whose asymptotic curves are cubics" by C. H. Sisam, 149–153; "Some two dimensional loci" by J. L. Walsh, 154–164.

TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY, volume 27, no. 3, July, 1925: "The linear complex of conics" by E. E. Libman, 265–269; "On the Weddle surface and

analogous loci" by Arnold Emch, 270–278; "Polynomials of several variables and their residue systems" by A. J. Kempner, 287–298; "A criterion for the conformal equivalence of a Riemann space to a Euclidean space" by Jesse Douglas, 299–306; "The deflection of a rectangular plate, fixed at the edges" by H. W. March, 207–217; "On irredundant sets of postulates" by Alonzo Church, 318–328; "Three dimensional manifolds of states of motion" by Harold Hotelling, 329–344; "Relations between the critical points of a real function of n independent variables" by Marston Morse, 345–396.

UNDERGRADUATE MATHEMATICS CLUBS.

All reports of club activities should be sent to H. J. ETTLINGER, 2910 Harris Park Ave., Austin, Texas.

CLUB TOPICS.

FIEDLER'S CYCLOGRAPHY.

By B. H. BROWN, Dartmouth College.

Fiedler [1] establishes a correspondence between the circles in a plane and the points of space as follows. At the center of any circle we erect a perpendicular to the plane of the circle on a specified side of the plane, which we shall call *above*, equal in length to the radius of the circle. The point at the end of this perpendicular shall "represent" the circle. Conversely, every point above the plane represents a circle. A point of the plane represents itself considered as a point circle.

In two articles E. Müller [2] has expressed his astonishment that so little interest has been shown in this extraordinarily instructive representation. This astonishment we share. Here is a representation which is real, natural, easily visualized, one where important systems of circles are represented by simple space loci and conversely; and yet this representation has been almost totally neglected while artificial, imaginary, but analytically perfect representations have absorbed all the attention of geometers. We shall here indicate some of the simplest applications and theorems; the reader may then turn to Fiedler, and to articles by Müller and Ogura.[3]

THEOREM 1. *The circles tangent to two intersecting half-lines in a plane are represented by the points of a half-line through the point of intersection of the half-lines, making with the plane an angle whose cotangent is the cosecant of half the angle between the half-lines.*

APPLICATION. THEOREM.[4] *The three external centers of similitude of three coplanar circles lie in a straight line.* PROOF. The plane determined by the three representing points of the circles will cut the plane of the circles in the straight line

[1] Fiedler, *Cyclographie*, Leipzig, 1882. *Die darstellende Geometrie*. See indexes, vols. 1 and 2.

[2] Müller, *Jahresbericht*, vol. 14, 1905; vol. 20, 1911.

[3] Ogura, *Tohoku Math. Journal*, vol. 3, 1913.

[4] *Cf.* Archibald, this MONTHLY (*1915*, 6). The method of Fiedler may be at once extended to a correspondence between the hyperspheres of $(n-1)$-space and the points of n-space; and the theorems in a short note by the author, this MONTHLY (*1916*, 155), follow immediately.

of the theorem. If we represent one of the circles by a point *below* the plane, we can show as easily the THEOREM: *The six centers of similitude of three coplanar circles lie by threes on four straight lines.*

THEOREM 2. *Conversely, the points of every half-line above the plane will be represented by circles tangent to two intersecting or parallel lines which will be real if the angle which the half-line makes with the plane is* $\leq \pi/4$.

COROLLARY 1. *The circles tangent to a line at a point, and which lie on one side of the line, are represented by the points of a half-line through this point, perpendicular to this line, and making an angle of* $\pi/4$ *with the plane.*

COROLLARY 2. *The circles of one system cutting a line under a constant angle* θ *are represented by points of a half-plane through this line.*

COROLLARY 3. *The circles of one system cutting a line under a constant angle* θ *at a given point are represented by the points of a half-line through this point, perpendicular to the common tangent to the circles, and making an angle* $\pi/4$ *with the plane.*

THEOREM 3. *The circles externally tangent to a given circle are represented by the points of a portion of a cone of revolution through the given circle.*

APPLICATION. PROBLEM OF APOLLONIUS: *To construct a circle tangent to three mutually external circles.* Analysis of the problem from Fiedler's point of view leads at once to the well-known construction of Gergonne. The student is referred to Fiedler [1] or to the very concise analysis of Coolidge.[2]

THEOREM 4. *The circles cutting a given circle under a fixed angle are represented by the points of a portion of an equilateral one-sheeted hyperboloid of revolution.*

PROOF: Let the fixed circle be of radius R and consider any circle of radius r cutting it under the fixed angle θ. Then if c is the distance between their centers,

$$c^2 = R^2 + r^2 - 2Rr\cos\theta.$$

Let x, y be a pair of rectangular axes in the plane, intersecting at the center of the given circle. Setting

$$c^2 = x^2 + y^2, \qquad r = z,$$

we have

$$x^2 + y^2 = R^2 + z^2 - 2Rz\cos\theta$$

as the equation of the locus of representing points.

APPLICATION. By considering Corollary 3 to Theorem 2 in connection with Theorem 4 we can discuss the rulings on the hyperboloid.

APPLICATION. STEINER'S PROBLEM: [3] *To construct a circle meeting three given circles at given angles.* The generalization from the solution of the problem of Apollonius is easy and natural.[4]

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[1] Fiedler, *Cyclographie, loc. cit.*, p. 30.
[2] *A Treatise on the Circle and the Sphere*, Oxford, 1916, p. 185.
[3] Steiner, *Crelle*, vol. 1, 1826, p. 162.
[4] Fiedler, *Cyclographie.*

CLUB ACTIVITIES.

MATHEMATICAL CLUB, TULANE UNIVERSITY, New Orleans, La.
[*1924*, 400.]

The Tulane Mathematical Club held three meetings during the year 1924–1925. The first was in November and the paper of the afternoon was on unified mathematics by Professor R. L. Menuet. The February meeting was addressed by Dr. C. G. Latimer on "The Geometrical Construction of the Complex Roots of a Cubic with Some Historical Remarks." The attendance was about forty at each of these meetings.

For the March meeting we had Professor F. R. Moulton of the University of Chicago who spoke on "Ballistics." This lecture was attended by about 100.

(Report by Professor H. E. Buchanan, Chairman.)

MATHEMATICS CLUB OF THE COLLEGE OF THE OZARKS, Clarksville, Arkansas.
[*1925*, 95.]

The Mathematics Club of the College of the Ozarks was organized for the year of 1924–25 as follows:

Pres., Annie Moon '26.
Vice-Pres., John Neal '27.
Sec.-Treas., Gladys Anderson '27.

Two members of the Club, Annie Moon and Gladys Anderson, were admitted as active members of "Chi Phi Mu" mathematical fraternity, the charter of which was recently granted to our mathematics department.

Meetings were held bi-monthly. Papers on the following topics were presented by members of the Club:

1. Einstein's theory of relativity.
2. Korzybski's concept of man.
3. Pedagogy of mathematics.
4. Mathematical philosophy.
5. Mathematical recreations.
6. History of mathematics.
7. Mathematical fallacies.
8. Geometry and faith.
9. Mathematics and religion.
10. Symmetry (general and applied).

(Report by Professor C. R. Hillard.)

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL, OTTO DUNKEL, AND H. L. OLSON.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo. All manuscripts should be typewritten, with double spacing and with a margin at least one inch wide on the left.

PROBLEMS FOR SOLUTION.

[N.B. Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known textbooks, or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

3153. Proposed by W. D. CAIRNS, Oberlin College.

Find the laws of motion, energy, etc., when the length of a simple pendulum is gradually shortened say by holding the thread between two fingers and drawing it up between them. Special cases: Draw up the thread (*a*) only at the ends of the swing; (*b*) at a uniform rate.

3154. Proposed by R. H. SCIOBERETI, Berkeley, California.

Find the equation of a spherical curve (Γ) such that if Q, P, N denote the intersections of a fixed great circle (C) with (1) the curve (Γ), (2) the great circle passing through a point M on the curve and orthogonal to the fixed circle (C), (3) the great circle normal to the curve at M respectively, then $QP = PN$, [the distances being measured along (C)].

EDITOR'S NOTE: Since to each point M on Γ correspond two points N and two points P, it follows that Γ must intersect C in two diametrically opposite points.

3155. Proposed by R. E. MORITZ, University of Washington.

Show that

$$\frac{d^n}{dx^n}\left[x^{m+n}(\log x)^n\right] = n!\,x^m\left[1 + {}_nP_1\log x + {}_nP_2\frac{(\log x)^2}{2!} + \cdots + {}_nP_n\frac{(\log x)^n}{n!}\right],$$

where ${}_nP_k$ denotes the sum of the products of the numbers $m+1, m+2, \cdots, m+n$, taken k at a time.

2676 [1918, 75]. Proposed by E. R. SMITH, State College, Pa.

Find the greatest term of the series

$$\frac{sp(sp-1)\cdots(sp-r+1)}{s(s-1)\cdots(s-r+1)}F(-r, -sq, sp-r+1, 1),$$

where s, r, sq and sp are positive integers, $r < s$, p and q are proper fractions such that $p+q=1$, and $F(-r, -qs, sp-r+1, 1)$ is a hypergeometric series. If s, r and $s-r$ are large, show that the greatest term is approximately equal to

$$\sqrt{\frac{s}{2\pi pqr(s-r)}}.$$

2775 [1919, 213]. Proposed by H. T. BURGESS, University of Wisconsin.

Solve in finite form, if possible, the differential equation

$$\frac{d^2y}{dt^2} + ay\frac{dy}{dt} + by = g,$$

where a and b are arbitrary constants and $g = 32.16$. When $t=0$, $y=0$, $dy/dt=0$.

2784 [1919, 366]. Proposed by T. H. GRONWALL, New York City.

Show that all solutions in integers of $y^2 = 1 + x + x^2 + x^3 + x^4$ are given by

$$\begin{array}{llll} x = -1, & 0, & 3; \\ y = \pm 1, & \pm 1, & \pm 11. \end{array}$$

2873 [1921, 36]. Proposed by D. H. RICHERT, Bethel College, Newton, Kan.

At B is the enemy's battery. At M_1 a battery is to be placed to silence B. Listening posts are installed at M_1, M_2, M_3, all provided with stop-watches. From the maps at hand, the three sides of the triangle $M_1M_2M_3$ are known. B is not visible from any one of the points M_1, M_2, M_3. The sound of a gun fired at B reaches M_1 at the time T, and M_2 at the time $T+\tau_1$ sec., and it reaches M_3 at the time $T+\tau_2$ sec. How far is B from M_1?

SOLUTIONS.

3115 [1925, 94]. Proposed by H. W. REDDICK, Cooper Union Institute of Technology.

Given a plane triangle and a system of coördinate axes in a 3-space, find the number of ways of placing the triangle so that its vertices lie on the axes.

SOLUTION BY FRED MILLER, Student, Cooper Union Institute of Technology.

Let the vertices of the given triangle be lettered A, B, C and its corresponding opposite sides a, b, c. Consider first the case in which a side lies upon an axis. If the triangle lies in the xy-plane with the side c on the x-axis and the vertex C on the y-axis, there are four possible positions, since C may be on the positive or negative y-axis and $c = AB$ may take the positive or negative direction along the x-axis. Four other positions are obtained by taking C on the x-axis and c on the y-axis. There are then eight positions in which c lies on one of the two axes in the xy-plane. Replacing in this reasoning c by a and b in turn, we find that there are 24 possible positions in the xy-plane. Considering all three planes, the total number of positions is 72, and this is true whether the triangle be acute, right or obtuse angled.

Next consider the case in which no side lies upon an axis, for example A on the x-axis, B on the y-axis, C on the z-axis, and denote the intercepts by x, y, z. Then $x^2 + y^2 = c^2$, $y^2 + z^2 = a^2$, $z^2 + x^2 = b^2$, and from these equations follow

$$x = \pm \sqrt{(b^2 + c^2 - a^2)/2} = \pm \sqrt{bc \cos A},$$

$$y = \pm \sqrt{(c^2 + a^2 - b^2)/2} = \pm \sqrt{ca \cos B},$$

$$z = \pm \sqrt{(a^2 + b^2 - c^2)/2} = \pm \sqrt{ab \cos C}.$$

Now if each angle of the triangle is acute, there are eight solutions for the intercepts considering the various combinations of $+$ and $-$ signs. But a, b, c may be permuted in each solution in 3! ways; there are therefore 48 possible positions.

If one angle is a right angle, the corresponding intercept is zero and the triangle must lie in a coördinate plane. This case has already been covered above. If one angle is obtuse, the corresponding intercept is imaginary and this position is impossible.

There are then for an acute angled triangle $72 + 48 = 120$ possible positions; for any other triangle there are 72 positions.

3116 [1925, 95]. Proposed by A. A. BENNETT, University of Texas.

Show that if a number is k times the number obtained by reversing the order of its digits, and neither has a zero for leading digit, then $k = 1, 4$, or 9. Show by example that 4 and 9 are actually possible.

SOLUTION BY HARRY LANGMAN, New York City.

The condition implies $k < 10$ and we shall suppose that $k > 1$. Let the unit's digit of the first number be a and the digit of the highest order, b. Then $kb \equiv a \pmod{10}$ and $ka \geqq b$. Hence if a is odd, neither k nor b can be even. With these restrictions the remaining possibilities are represented in the following table which is explained below:

k	a	b	a	Result
2	2	4 5	8 0	x x
	4	8 9	6 8	x x
3	1	3 5	9 5	x x
	2	6 7 8	8 1 4	x x x
	3	9	7	x
4	2	8 9	2 6	? x
5	1		0 5	x x
6, 8	1		even	x
7	1	7 9	9 3	x x
9	1	9	1	?

Here if $k = 2$, for example, a (second column) is restricted by the condition $2a < 10$. Hence $a = 1, 2, 3$ or 4, but as a must be even, 1 and 3 are omitted. The left-hand digit of the first number, b, is clearly ka plus what may be carried—at most $k - 1$. For $k = 2$, the number carried is either 0 or 1. Hence for $a = 2$ the only possibilities for b are 4 and 5, and for $a = 4$ they are 8 and 9 (third column). But the unit's digit of the first number, a, is obtained from kb. Hence for $k = 2$ and $b = 4$ or 5 the corresponding possibilities for a (fourth column) are 8 and 0, in each case contradicting the value in column 2. This is indicated by the "x" in the last column.

This shows that the only possibilities for k, omitting $k = 1$, are $k = 4$ and $k = 9$. In

the first case $a = 2$ and $b = 8$; in the second, $a = 1$ and $b = 9$. Using the method of skeleton division, it is easily seen that for $k = 4$ the only numbers possible are 8712, 87912, 879912, $\cdots$, 879$\cdots$912, $\cdots$, and for $k = 9$ they are 9801, 98901, 989901, $\cdots$, 989$\cdots$901, $\cdots$. These numbers are all multiples of 99.

EDITOR'S NOTE: It should be observed that 87128712, 871287128712, 98019801, etc., also are solutions.

Also solved by W. A. REES.

3117 [1925, 95]. Proposed by W. J. SIDIS, New York City.

It is well known that, except for a possible factor 3, all prime factors of $m^2 \pm mn + n^2$ must be of the form $6k + 1$, if m and n are prime to each other. Prove that, if n is an odd prime, all prime factors of

$$x^{n-1} \pm x^{n-2}y + \cdots \pm xy^{n-2} + y^{n-1},$$

excluding a possible factor n, must be of the form $2kn + 1$, provided x and y are prime to each other.

SOLUTION BY HARRY LANGMAN, New York City.

In the given expressions the terms must be either all positive or alternately positive and negative; let us call these cases "first" and "second."

Let p be a prime factor of the first expression, $A(x, y)$. Then both x and y are prime to p. We have

$$(x - y)A = x^n - y^n \equiv 0 \pmod{p}.$$

We have also

$$x^{p-1} \equiv y^{p-1} \equiv 1 \pmod{p}.$$

Suppose $p - 1$ not a multiple of n. Then $p - 1$ is prime to n, and we may write $\alpha(p - 1) - \beta n = 1$. But $x^{\alpha(p-1)} \equiv y^{\alpha(p-1)} \pmod{p}$, from which $x^{\beta n+1} \equiv y^{\beta n+1}$, giving $x \equiv y \pmod{p}$. If we assume $x - y$ prime to p, we must have $p - 1$ a multiple of n, and, as n is odd, and p is odd, this multiple must be even, giving $p = 2kn + 1$. If $x - y$ is a multiple of p, we may put $x = y + mp$, from which

$$A(x, y) = A(y + mp, y) \equiv A(y, y) \equiv ny^{n-1} \equiv n \equiv 0 \pmod{p},$$

requiring $p = n$.

In the second polynomial, $A(x, -y)$, let $-y \equiv y' \pmod{p}$, and put $-y = y' - mp$. Then

$$A(x, -y) = A(x, y' - mp) \equiv A(x, y').$$

This is in the first form and the theorem follows.

3118 [1925, 95]. Proposed by HARRY LANGMAN, New York City.

If $n > 2$ and ϵ is a primitive root of $\epsilon^n = 1$, show that

$$\begin{vmatrix} \epsilon & \epsilon^2 & \epsilon^3 & \cdots & \epsilon^{n-1} \\ \epsilon^2 & \epsilon^4 & \epsilon^6 & \cdots & \epsilon^{2(n-1)} \\ \epsilon^3 & \epsilon^6 & \epsilon^9 & \cdots & \epsilon^{3(n-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \epsilon^{n-1} & \epsilon^{2(n-1)} & \epsilon^{3(n-1)} & \cdots & \epsilon^{(n-1)(n-1)} \end{vmatrix} = (-1)^{(n-1)(n-2)/4} n^{(n-2)/2}.$$

SOLUTION BY J. J. NASSAU, Case School of Applied Science.

Let us write e instead of ϵ.

It is clear that

$$\begin{vmatrix} e & e^2 & e^3 & \cdots & e^{n-1} \\ e^2 & e^4 & e^6 & \cdots & e^{2(n-1)} \\ e^3 & e^6 & e^9 & \cdots & e^{3(n-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ e^{n-1} & e^{2(n-1)} & e^{3(n-1)} & \cdots & e^{(n-1)(n-1)} \end{vmatrix}$$

$$= e \cdot e^2 \cdot e^3 \cdots e^{n-1} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ e & e^2 & e^3 & \cdots & e^{n-1} \\ e^2 & e^4 & e^6 & \cdots & e^{2(n-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ e^{n-2} & e^{2(n-2)} & e^{3(n-2)} & \cdots & e^{(n-2)(n-1)} \end{vmatrix}.$$

The coefficient of this determinant being equal to $e^{n(n-1)/2}$ or 1, we have the original determinant equal to the well-known difference-product

$$\zeta^{1/2}(e, e^2, e^3, \cdots, e^{n-1}).$$

But

$$\zeta(e, e^2, e^3, \cdots, e^{n-1}) = \begin{vmatrix} S_0 & S_1 & S_2 & \cdot & \cdot & \cdot & S_{n-2} \\ S_1 & S_2 & S_3 & \cdot & \cdot & \cdot & S_{n-1} \\ S_2 & S_3 & S_4 & \cdot & \cdot & \cdot & S_n \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{n-2} & S_{n-1} & S_n & \cdot & \cdot & \cdot & S_{2(n-2)} \end{vmatrix},$$

where $S_i = (e)^i + (e^2)^i + \cdots + (e^{n-1})^i$ (Scott and Mathews, *Theory of Determinants*, second edition, page 152).

Making use of Newton's formulæ for sums of powers of roots (Burnside and Panton, *Theory of Equations*, second edition, page 290), we have for the equation $x^n - 1 = 0$,

$$\begin{aligned} S_0 &= (e)^0 + (e^2)^0 + \cdots + (e^{n-1})^0 = -(e^n)^0 + n = n - 1, \\ S_1 &= e + e^2 + \cdots + e^{n-1} = -e^n = -1, \\ S_2 &= (e)^2 + (e^2)^2 + \cdots + (e^{n-1})^2 = -(e^n)^2 = -1, \\ &\cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \\ S_n &= n - 1, \\ S_{n+1} &= -1. \end{aligned}$$

Therefore

$$\zeta(e, e^2, e^3, \cdots, e^{n-1}) = \begin{vmatrix} n-1 & -1 & -1 & -1 & \cdot & \cdot & \cdot & -1 & -1 \\ -1 & -1 & -1 & -1 & \cdot & \cdot & \cdot & -1 & -1 \\ -1 & -1 & -1 & -1 & \cdot & \cdot & \cdot & -1 & n-1 \\ -1 & -1 & -1 & -1 & \cdot & \cdot & \cdot & n-1 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -1 & -1 & -1 & n-1 & \cdot & \cdot & \cdot & -1 & -1 \\ -1 & -1 & n-1 & -1 & \cdot & \cdot & \cdot & -1 & -1 \end{vmatrix}.$$

Subtracting the second row from the first, the third, fourth, etc., we have

$$\zeta(e, e^2, e^3, \cdots, e^{n-1}) = \begin{vmatrix} n & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ -1 & -1 & -1 & -1 & \cdot & \cdot & \cdot & -1 & -1 \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & n \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & n & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & n & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & 0 & n & 0 & \cdot & \cdot & \cdot & 0 & 0 \end{vmatrix} = (-1)^{(n-1)(n-2)/2} n^{n-2}.$$

By extracting the square root of both sides the proposition is established.

A more general form of the given determinant appears in the *Annals of Mathematics*, second series, vol. 28, nos. 1 and 2, page 111.

Also solved by THEODORE BENNETT who sent in two solutions, each differing from the other and from the one above, and by W. A. REES.

NOTES AND NEWS.

Readers are invited to contribute to the general interest of this department by sending items to H. W. KUHN, Ohio State University, Columbus, Ohio.

Professor E. V. HUNTINGTON of Harvard University has recently given a series of twelve lectures at Beloit College on the subject "The Logical Foundations of Elementary Mathematics."

Associate Professor J. W. CAMPBELL of the University of Alberta has been promoted to a full professorship of mathematics.

Professor ETHELWYNN R. BECKWITH, who was acting head of the department of mathematics at the College for Women at Western Reserve University last year, has been appointed head of the department of mathematics at Milwaukee-Downer College.

Mr. A. B. LEWIS of the University of Mississippi has been promoted to an assistant professorship of mathematics.

Professor C. D. SMITH has resumed the duties of head of the department of mathematics at Louisiana College after a year's leave of absence spent at the University of Iowa.

Miss JEANNETTE RYNO has been appointed to an assistant professorship of mathematics at Lander College, Greenwood, S. C.

Mr. F. A. PARISI has been appointed to an assistant professorship of mathematics at the College of St. Teresa.

Mr. D. H. RICHERT of Bethel College has been promoted to a full professorship of mathematics.

Professor H. W. TAYLOR, formerly professor of mathematics at Emporia College, has been appointed professor of mathematics at Southwestern College.

At the University of Oklahoma, Professor S. W. REAVES, who for the past two years has been acting dean of the College of Arts and Sciences, has been appointed dean. Associate Professor E. D. MEACHAM has been promoted to a full professorship of mathematics. Associate Professor NATHAN ALTSHILLER-COURT has resumed his duties after a leave of absence for one year spent in Paris. Miss DORA MCFARLAND has returned to her duties as instructor of mathematics after spending a year in study at the University of Chicago. Dr. ELSIE J. MCFARLAND, formerly of the University of California, Mr. S. B. TOWNES, and Mr. J. C. BRIXEY have been appointed instructors in mathematics.

At the University of Georgia, Professor C. M. SNELLING, Dean and Professor of Mathematics, has been elected Acting Chancellor and Professor R. P. STEPHENS of the same department has been elected Acting Dean. Mr. FORREST CUMMING has been promoted to an adjunct professorship of mathematics and E. M. EVERETT has been appointed to an instructorship. Professor D. F. BARROW of the department of mathematics has been elected Secretary of the Georgia Academy of Science.

At the Mississippi Agricultural and Mechanical College, Dr. B. M. WALKER, formerly head of the department of mathematics, has been elected president of the college. Associate Professor H. FOX has been made professor and head of the department of mathematics. Mr. C. R. STARK has been appointed to an associate professorship of mathematics and Mr. A. EDMONDSON has been appointed to an instructorship.

At Bowdoin College, Professor W. A. MOODY is on leave of absence for the year 1925–1926. Assistant Professor E. S. HAMMOND has been made professor and acting head of the department for the current year. Mr. C. T. HOLMES has been appointed to an assistant professorship of mathematics and Mr. ROY HALE to an instructorship.

At Western College for Women, Dr. HELEN TAPPAN, formerly associate professor of mathematics at Iowa State College, has been appointed professor and head of the department of mathematics. Miss RUTH DEWEY has been appointed instructor.

At the University of Wyoming, Mr. O. H. RECHARD has been promoted to associate professorship of mathematics. Miss GRETA NEWBAUER has been appointed to an instructorship and Mrs. R. H. HARRIS and Miss MARIAN PRATER have been appointed student instructors of mathematics.

At Georgia Wesleyan College, Professor J. C. HINTON has retired as head of the department of mathematics and astronomy after thirty-five years of service. Professor FREDERICK WOOD of Lake Forest College has been elected to succeed him. Miss RUTH LEONARD has been appointed to an instructorship in mathematics.

The following appointments to instructorships of mathematics are announced:

Case School of Applied Science, Mr. P. D. WILKINS, formerly instructor in mathematics at Tufts College.

Goucher College, Dr. MARIAN M. TORREY, formerly instructor in mathematics at the University of Illinois.

Kansas State Agricultural College, Mr. R. C. STALEY.

Otterbein College, Mr. H. E. MENKE.

South Dakota State College, Mr. G. D. GORE.

Toledo University, Mr. J. B. WINSLOW.

Union College, Mr. E. W. POWELL.

Wheaton College, Miss KATHLEEN SEARS.

Atlanta University, Mr. CLEMENT SUTTON.

Beloit College, Mr. L. COTHERN.

Professor J. A. MILLER, Associate Professor R. W. MARRIOTT, and Mr. D. B. MCLAUGHLIN, instructor in mathematics, of Swarthmore College have gone to Sumatra in order to observe the total solar eclipse of January 14, 1926.

At the Virginia Polytechnic Institute, Blacksburg, Va., Professor J. E. WILLIAMS, head of the department of mathematics, has been promoted to dean

of the college. He still continues as head of this department. At its June commencement Hampden-Sidney College conferred the degree of LL.D. upon Dr. WILLIAMS.

At the same institution, the following instructors in the department of mathematics have been promoted to assistant professorships: T. W. HATCHER, H. C. AHALT, A. V. MORRIS, and F. S. GLASSETT.

Dr. F. S. NOTESTEIN, for many years Professor of Mathematics and Astronomy at Alma College, died July 22, 1924, after a short illness.

Professor JOHN PHILLIPS of the department of mathematics, Kansas University, died May 25, 1925.

The Mathematical Association of America announces that the second of the Carus Mathematical Monographs is ready for the printer. It is expected that it will be ready for distribution in January, 1926, on the same plan as the first monograph was distributed in 1925. The title is *Functions of a Complex Variable* by D. R. CURTISS, of Northwestern University. It will be a volume of the same style and of approximately the same number of pages as the first monograph the sale of which, both to members of the Association and to the general public, has been gratifying.

At Yale University, the Sterling fellowships have been established by a gift of one million dollars from the trustees of the estate of the late JOHN W. STERLING to stimulate scholarship and advanced research in all fields of knowledge. They are open equally to graduates of Yale University and other approved colleges and universities in the United States and foreign countries, to both men and women, whether graduate students, or instructors or professors when on leave of absence, who desire to carry on studies and investigations under the direction of the graduate faculty of Yale University or in affiliation with that body.

The Sterling fellowships are divided into two general classes: *research or senior fellowships* and *junior fellowships*. Candidates for research or senior fellowships must have the Ph.D. degree, or must have had such training and experience in research as are indicated by this degree. Candidates for junior fellowships must be well advanced in their work towards the Ph.D. degree. In exceptional circumstances, holders of either class of fellowships, who have been in residence at Yale University for a year or more, may be permitted to carry on their investigations in part elsewhere, at home or abroad. The Sterling fellowships are awarded on the understanding that the recipients shall not engage in teaching during the tenure of appointment.

The stipends of the research or senior fellowships range from $1,000 to $2,500 or more, dependent upon the character of the proposed investigation. The

stipends of the junior fellowships range from $1,000 to $1,500. For special purposes, such as carrying through to completion a piece of investigation, awards may be made of less than $1,000. Fellows who have not yet obtained the Ph.D. degree are subject to the usual tuition and laboratory fees. All fellows are appointed for a single year, but may be reappointed with or without additional stipend.

Holders of Sterling fellowships are required to submit reports on their work, either at stated intervals or at the expiration of their fellowships; and when the results of their work are published they are expected to give proper credit to the assistance they have received as Sterling fellows.

Applications for these fellowships should be addressed to the Dean of the Graduate School of Yale University, New Haven, Connecticut, on blanks which may be obtained from him. Applications for the junior fellowships must be submitted by March 1, and applications for the senior fellowships by April 1.

INDEX TO VOLUME XXXII, 1925.

By H. S. EVERETT, Bucknell University.

Misprinted names in the text are corrected in this index.

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ADDENDA AND CORRIGENDA.

P. 71, l. 9, *for* "$+\cdots)^2\}^{1/2}$" *read* "$+\cdots)^2)\}^{1/2}$."
P. 71, last line, *for* "μ_s/σ_x^5" *read* "μ_5/σ_x^5."
P. 72, lines 3 and 5, *for* "$(\beta_4 - \beta_2 - \beta_1)$" *read* "$(\beta_4 - \beta_2^2 - \beta_1)$."
P. 72, line 10, numerator, for "$+\frac{2}{3}$" read "$+\frac{1}{3}$."
P. 162, **Theorem,** *delete* "*such that* $0 < f(x_1)/[f(x_1) - f(x_n)] \leqq 1$."
P. 388, above third display, *for* "transformed determinant" *read* "original determinant."
P. 388, below third display, *for* "*Journal* ·····, p. 47." *read* "*Transactions of the American Mathematical Society*, vol. 26, 1924, p. 113."

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